

# PRODUCTION MODEL DESCRIPTION\*

Simple Production Model with Vertical Interaction

ProdMod5

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## Abstract

In this document we present a ‘wide’ description of the model ProdMod5. The model is a computational representation (semi agent-based) of an industrial organisation, describing a complex firm production process with vertical interactions between sectors in the acquisition and selling of inputs/outputs. The aim is to provide an understanding of the model at various levels. We start with a general qualitative description of the entire process, and some of the underneath rationale. Contemporaneously we show the object structure of the model (where objects stand for ‘complex variables’). Finally, for the interested reader we provide the specific functional construction, with reference to the object structure and the computational implementation.

**Keywords:**

**JEL-classification:**

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## 1 Model Overview

This section provides an overview of the model describing the main elements and dynamics in very general terms. The first paragraph describes the data structure of the model, indicating the model's labels of the objects representing the entities involved in the simulation. The second paragraph describes briefly the main dynamics of the model.

### 1.1 Data structure

□

Figure 1: Object structure of the model. The structure is composed by a hierarchical tree. Upper objects contain one or more copies of the lower types of objects, while each lower object is contained in one single copy of an upper objects.

The model represents an economic system composed by several sectors (**Market**) each producing one product defined over a set of characteristics. Sectors record different statistics, like total production, average price, average quality for each of the characteristic (**Characteristic**). In each sector there are several firms (**Firm**), each producing one single product, whose output is defined both in terms of quantity and in terms of the qualities measured over the product's characteristics (**OutCh**). Firms record financial data (e.g. revenues, profits, etc.) and production data (e.g. stock levels). Firms use production inputs (**Input**) defined over several quality dimensions (**InCh**). Each input is assigned a technical coefficient indicating how many units of that input are necessary for each unit of the firm's output. Firms have different competences in producing each of their own output characteristics (**Competence**). Each of such competences is specific for one input (**InComp**) and one characteristic of such input (**CompInCh**).

### 1.2 Model Dynamics

The model dynamics represents production processes carried out by firms. The pace of a simulation run is such that each step represents the time requested to perform small changes in production levels, say the equivalent of a week in an manufacturing industries. Many of the variables dynamics therefore are expressed as smoothed changes. For example, a variable  $X$  computes at each time  $t$  a target, or desired, level  $X_t^*$  but its actual value is  $X_t = X_{t-1}\alpha + (1 - \alpha)X_t^*$ , moving only partially form its current level in the direction of the desired level.

#### 1.2.1 Production

Firms can serve both a partly endogenous final demand and the demand from firms in other sectors (business demand). Whether a firm receives one or both demand types is exogenously determined at sector level. Final demand is computed in two steps: firstly the total amount of the final demand is assessed as a function of the average price and average product characteristics at sector level; secondly, individual demand from final consumers is computed as a function of the relative quality of a firm's product in respect of competitors' in the same level.

Business demand for a firm from firms in other sectors is implemented as the direct orders from buying firms to the specific supplier firm (see below for changes in the supplying firms). Firms determine the level of production aiming both at satisfying the expected demand and to maintain stocks at a desired level. Firms firstly determine the desired

production levels considering their current final demand (if any), business demand from the previous period (if any) and desired adjustment of stocks. Secondly, they dispatch purchasing orders to their suppliers (if any: sales of exogenous inputs are not recorded but as costs of the purchasing firms) and carry on their production. The purchasing orders are determined by the technical coefficients multiplied by the desired production. Subsequently, all the orders are fulfilled, though, in some cases, this may entail undesired stock variations.

During the production process firms determine the quality of their output for each characteristic, as the result of their competences and the quality levels of the inputs.

### 1.2.2 Pricing and Financial Records

The price of a product is undifferentiated for final consumers and intermediate firms. The price is determined using a mark-up over the average costs of production. Such costs are determined computing the total costs of production as the amount of inputs needed times the prices of the inputs. Firms record the revenues, profits and financial capital, computed as the profits cumulated through time.

### 1.2.3 Suppliers' Selection

Firms in the same sector need the same types of inputs. The model admits exogenous inputs, whose values are assigned in the beginning of the simulation and never modified, and inputs from other sector in the model. For this second class of inputs each firm selects a specific firm on the input sector in the beginning of the model. Every given number of periods a client firm review its suppliers. A new firm from the supplier sector is selected randomly with probabilities proportional to an indicator of the quality of the input (including the price). When a new supplier is selected the client firm replace the old quality coefficients of its input with those of the new supplier.

## 2 Production process and I/O relations

### 2.1 Getting started

Where not specified, period is  $t=0$  and all firms pertain to the same sector of production. There is only one final general market and no single buyers that directly relate with producers. Inputs not supplied by firms of other sectors are bought from general input markets (in which price and qualities are defined).

Each firm  $i$  pertains to a finite population  $I = \{1, \dots, n\}$ , divided in subpopulations of  $i_S \in S_{F-s_j} = \{1, \dots, n_S\}$  (and  $i_S \notin S_{F-s_i}$  if  $i \neq j$ ), where  $S$  is the sector to which it pertains<sup>1</sup> and  $s_j = \{0, \dots, s_{max}\}$  indicates the sectors ordination from the final to the primary one ( $s_{max} > 0$ )<sup>2</sup>. In sum, the overall economy i) is formed by  $n$  firms; ii) each firm pertains to only one sector; iii) the sum of sectors form the 'global' economy; and iv) sectors are ordered in an abstract way from the first supplier to the final market. Each firm produces a good which is homogeneous in terms of final usage, but heterogeneous in terms of price and quality features, i.e. in the utility of final use, in a Lancasterian fashion (Lancaster, 1966). Hence, output goods for each firm  $i$  in each sector  $S$  are represented as vectors of quality characteristics (Saviotti and Metcalfe, 1984):

$$\vec{y}_{i_S} = \{y_{1,i_S}, y_{2,i_S}, \dots, y_{m,i_S}\}$$

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<sup>1</sup>Where each  $S_{F-s} \subset I$ , and  $\sum_{s=0}^{s_{max}} S_{F-s} \subseteq I$

<sup>2</sup>Pedix  $F$  indicates the final sector, and  $F - s_{max}$  the first one

That is, one unit of good produced by the  $i^{th}$  firm is defined by a vector of  $m$  values, where  $m = \{1, \dots, M\}$ , one for each characteristic<sup>3</sup>.

The production process (of each unit) takes place by combining inputs and firm-specific competencies, following the framework drawn by Saviotti and Metcalfe (1984), and modified by Gallouj and Weinstein (1997). Each firm thus has a certain amount of initial resources (material and immaterial) and combines them to obtain the final product.

## 2.2 Production process

### 2.2.1 Output — $y_m$ ( $y$ )

The production of one single output requires the use of different inputs (e.g. intermediate, primary and capital), each of which can be introduced in the production process provided the existence of given competencies<sup>4</sup>. We thus adapt the Saviotti-Metcalfe framework depicting a set of  $k = \{1, \dots, K\}$  input vectors  $\vec{w}_{i,k} = \{w_{1,i,k}, w_{2,i,k}, \dots, w_{h_k,i,k}\}$  of  $h_k = \{1, \dots, H_k\}$  inputs' features<sup>5</sup>, which can differ (in both numerosity and value) for each input  $k$  used for the production of a good. Similarly, we construct a vector  $\vec{A}$ , of which each element  $m = \{1, \dots, M\}$  is a set of  $K$  vectors with  $H_k$  elements, provides the values of the competencies  $a_{m,h_k,k} = \left\{1, \dots, \sum_{k=1}^K \sum_{h_k=1}^{H_k} h_k \cdot M\right\}$  acquired for the use of each specific combination of input and their features, and defines one product feature combining with  $\vec{w}_k \times K$ . The result for one single feature of output is then formally represented in the following way<sup>6</sup>:

Contained in object **OutCh** → **Output** → **Firm** → **Market**

For each firm  $i$

Value of each quality feature as the output of the production process

Elements used:

Element	Description	In Object	LSD
$a_{m,h_k,k}$ (P)	firm competencies attached to each input quality feature, for the number of features of the specific input, for the specific output quality feature	CompInCh	a
$w_{h_k,k}$ (P)	quality features of each input used in the production	InCh	w
$h_k$ (P)	number of feature for each input	InCh	n.d.

$$y_m = 1 + \frac{\sum_{k=1}^K \sum_{h_k=1}^{H_k} a_{m,h_k,k} \cdot w_{h_k,k}}{\sum_{k=1}^K \sum_{h_k=1}^{H_k} h_k} \quad (1)$$

<sup>3</sup>One can think at any characteristic that can be attached to a good, both sector, product, firm, ... dependent or independent (e.g. cheapness, standardisation, marketing, etc.)

<sup>4</sup>In this model version we have not accounted for the relation between skills and input features, but it is quite straightforward, technically, to introduce threshold values for competencies.

<sup>5</sup>Different inputs can have a different number of characteristics. Inputs are thus represented in the same form of the output

<sup>6</sup>For the sake of graphical simplification we suppress firm, time and sector indexes. In the rest of the document they will be introduced only when necessary

Thus,  $y_m$  is primarily an output of the firm-level *general* competences  $m$ , of the *single characteristic* of each input, and the associated competence to each characteristic, and/or each input. This kind of modelling allows for a number of changes and combinations in the use of competences, which can vary for all inputs (e.g. general learning by doing at the firm level, or change in the organisation, etc.), for specific ones (e.g. increase in capital input quality, imports of specific materials, etc.), or one of their features.

### 2.2.2 Inputs quantity — $q_k^I$ (Tot)

Contained in object Input  $\rightarrow$  Firm  $\rightarrow$  Market

For each firm  $i$  and input  $k$

Quantity of input needed for each output unit, given by a technical coefficient

Elements used:

Element	Description	In Object	LSD
$q$ (V)	quantity of output produced	Firm	Quantity
$\beta_k$ (P)	quantity coefficient	Input	quantity_coeff

$$q_k^I = \beta_k \cdot q \quad (2)$$

## 2.3 Financial accounts

### 2.3.1 Variable costs — $c^V$ (VariableCosts)

Contained in object Firm  $\rightarrow$  Market

For each firm  $i$

Total costs for variable inputs

Elements used:

Element	Description	In Object	LSD
$q_k^I$ (V)	quantity of inputs used	Input	Tot
$p_k^I$ (P, V)	price of each input k, initially given (when the input is bought on a general input market; else, given by the output price of the supplier firm)	Input	price_input

$$c^V = \sum_{k=1}^K q_k^I \cdot p_k^I \quad (3)$$

### 2.3.2 Output price — $p$ (Price)

Contained in object Firm  $\rightarrow$  Market

For each firm  $i$

Price of the final good as a mark-up decision of the single firm.

Elements used:

Element	Description	In Object	LSD
$mkp$ (P)	mark-up applied to total variable costs. (initially given, has to be endogenized in the firm decision as a function of input price, quality feature and market share change)	Firm	mark-up
$c^V$ (V)	variable costs	Firm	VariableCosts
$q$ (V)	quantity produced by each firm, used to obtain the unit variable cost	Firm	Quantity

$$p = \begin{cases} \frac{c^V}{q} (1 + mkp) & \text{if } q > 0 \\ \sum_{k=1}^K p_k^I \cdot \beta_k & \text{if } q = 0 \end{cases} \quad (4)$$

### 2.3.3 Revenues — $R$ (Revenues)

Contained in object Firm  $\rightarrow$  Market

For each firm  $i$

Total revenues from sales

Elements used:

Element	Description	In Object	LSD
$Y$ (V)	quantity sold by the firm (different from the quantity produced, depending on sectors)	Firm	Sales
$p$ (V)	price of output	Firm	Price

$$R = Y \cdot p \quad (5)$$

### 2.3.4 Final sales — $Y^{S_F}$ (FinalSales)

Contained in object Firm  $\rightarrow$  Market

For each firm  $i$  in the final market (or selling also to a final market)

Total quantity sold to the final market

Elements used:

Element	Description	In Object	LSD
$ms$ (V)	firm's market share (in the final market)	Firm	Ms
$D$ (V)	demand in the final market	Market	Demand

$$Y^{S_F} = \begin{cases} ms \cdot D & \text{if sector} = S_F \\ 0 & \text{if sector} = S_{F-s_j, s \neq 0} \end{cases} \quad (6)$$

### 2.3.5 Total Sales — $Y$ (Sales)

Contained in object Firm  $\rightarrow$  Market

For each firm  $i$

Computes the value of the total sales that is also the difference between the production of the current year plus previous year stock and current year stock.

Elements used:

Element	Description	In Object	LSD
$OB$ (V)	Order book, i.e. the quantity requested by client firms	Firm	OrderBook
$Y^{SF}$ (V)	sales in the final market	Firm	FinalSales

$$Y = OB + Y^{SF} \quad (7)$$

### 2.3.6 Profits — $\pi$ (Profit)

Contained in object Firm  $\rightarrow$  Market

For each firm  $i$

Provides the final profits, given also the fixed costs (which for assumption do not enter in mark-up decision)

Elements used:

Element	Description	In Object	LSD
$R$ (V)	firm revenues	Firm	Revenues
$c^V$ (V)	variable costs	Firm	VariableCosts
$c^F$ (P)	fixed costs, initially given	Firm	FixedCosts

$$\pi = R - c^V - c^F \quad (8)$$

## 2.4 Demand and market dynamic

### 2.4.1 Average output price — $\bar{p}_{t,S}$ (AvPrice)

Contained in object Market

Equal for all the firms at each time  $t$  in each sector  $S$

Provides the value of the average price of all firms, as it is the information on which consumers rely to make their consumption decision (given a parameter of sensitivity); thus, in period  $t$  it refers to information held by consumer in period  $t-1$ . The average is weighted by each firm produced quantity (which is proportional to the market share).

Elements used:

Element	Description	In Object	LSD
$p_t$ (V)	price of the output in the current period	Firm	Price
$ms_{t-1}$ (V-1)	market share of each firm in period $t-1$	Firm	Ms

$$\bar{p}_{t,S} = \frac{\sum_{i_S=1}^{n_S} p_{i_S,t} \cdot ms_{i_S,t-1}}{\sum_{i_S=1}^{n_S} ms_{i_S,t-1}} \quad (9)$$

**2.4.2 Average quality by feature** —  $\bar{q}_{m,t,s}$  (AvValue (in AvPrice))

Contained in object **Characteristic** → **Market**

Equal for all firms  $i$  but different for each feature  $m$  at time  $t$  and in sector  $S$

As for the price, average quality for each feature  $m$  determines the final demand (given a parameter of sensitivity for each characteristic).

Elements used:

Element	Description	In Object	LSD
$y_{m,t}$ (V)	quality of the feature $m$ of the output in the current period	OutCh	y
$q_{t-1}$ (V-1)	output quantity in period $t-1$	Firm	Quantity

$$\bar{q}_{m,t,s} = \frac{\sum_{i_S=1}^{n_S} y_{i_S,m,t} \cdot m s_{i_S,t-1}}{\sum_{i_S=1}^{n_S} m s_{i_S,t-1}} \quad (10)$$

**2.4.3 Target demand** —  $D^*$  (TargetDemand)

Contained in object **Market**

Equal for all firms  $i$  in the final sector at each time  $t$

Determines the *theoretical* demand as a function of price and quality, which is not yet the demand directly faced by firms, but requires consumers to adapt to good's features. This demand function is used by firms in the final sector, as firms in the previous ones have their demand determined by the buyers.

Elements used:

Element	Description	In Object	LSD
$H$ (P)	constant	Market	Constant
$\bar{p}_{t-1}$ (V-1)	average price of the final sector	Market	AvPrice
$\bar{y}_m$ (V)	average values of the quality $m$ features	Characteristic	AvValue
$\alpha^p$ (P)	parameter of sensitivity to price	Market	alphaPrice
$\alpha_m^y$ (P)	parameter of sensitivity to quality features (one for each $m$ feature of the good)	Characteristic	alpha

$$D^* = H \cdot (1/\bar{p}_{t-1})^{\alpha^p} \prod_{m=1}^M \bar{y}_m^{\alpha_m^y} \quad (11)$$

**2.4.4 Demand** —  $D$  (Demand)

Contained in object **Market**

Equal for all firms  $i$  in each sector  $S$

Computes the actual demand faced by all firms at time  $t$ . For firm in the final market is given by the smooth adjustment to target demand, while it is the only 'valid' demand function for other firms (for firms belonging to the intermediate sectors is given by the needs of the final sector, when no alternative, final market is introduced). Thus, for the final mkt. the equation represents the demand side adjustment of consumers to the different goods produced by firms.

Elements used:

Element	Description	In Object	LSD
$D^*$ (V)	target demand	Market	TargetDemand
$D_{t-1}$ (V-1)	lagged demand	Market	Demand
$s^D$ (P)	smooth parameter for the demand	Market	smoothDemand
$OB$ (V)	a parameter in the computational model, which act as a variable, being overwritten in each period: provides the quantity of inputs ordered by buyers	Firm	OrderBook

$$D = \begin{cases} D_{t,S_F} = s^D D_{t-1} + (1 - s^D) D_t^* & \text{if sector} = S_F \\ D_{t,S_{F-s_j}} = \sum_{i=1}^{n_S} OB_{i,S_{F-s_j}} & \text{if sector} = S_{F-s_j, s_j \neq 0} \end{cases} \quad (12)$$

The two demands have an inverse formation process: while in the final sector we compute first the amount of demanded goods, and the sum of firms' quantities will tend to equal them, for the intermediate sectors the demand is computed after the quantities and reflects their sum (is then assumed to be equal to the quantity produced, i.e. demanded by following sector)

## 2.5 Firm's position

### 2.5.1 Competitiveness index — $I$ (IndexQ)

Contained in object **Firm** → **Market**

Equal for all firms  $i$

Provides an index of competitiveness of each firm, which is related to the final demand parameters (thus a market competitiveness)

Elements used:

Element	Description	In Object	LSD
$p_{t-1}$ (V)	price of firm output in the previous period	Firm	Price
$y_m$ (V)	output features	OutCh	y
$\alpha^p$ (P)	parameter of sensitivity to price	Market	alphaPrice
$\alpha_m^y$ (P)	parameter of sensitivity to quality features (one for each $m$ feature of the good)	Characteristic	alpha

$$I = (1/p_{t-1})^{\alpha^p} \prod_{m=1}^M y_m^{\alpha_m^y} \quad (13)$$

### 2.5.2 Target market share — $ms^*$ (TargetMs)

Contained in object **Firm** → **Market**

For all firms  $i$ , in the final sector

Computes the *theoretical* market share, which each firm (in the final sector) would have if consumers perfectly new the conditions of the market. As for the target demand equation, it is not the market share really faced by firms at the beginning of the production cycle, and is applicable only to firms selling in a final market.

Elements used:

Element	Description	In Object	LSD
$I$ (V)	firms' competitiveness index	Firm	IndexQ

$$ms^* = \begin{cases} \frac{I}{n_S} & \text{if sector} = S_F \\ \sum_{i=1} I_i & \\ ms & \text{if sector} = S_{F-s_j, s \neq 0} \end{cases} \quad (14)$$

We implicitly assume that the firms are more able than consumers to recognise the feature of suppliers and then place order on them (no adjustment in the  $ms$  of providers from 'theoretical' to real). To introduce uncertainty or 'error' in the choice of the inputs we use the order of inputs equation.

### 2.5.3 Market share — $ms$ (Ms)

Contained in object **Firm** → **Market**

For each firm  $i$  at time  $t$ , depending on the sector  $S$

As for the demand, in the final market the actual market share adjusts through time, as consumers get accustomed to the goods sold. While for intermediary sectors it is deterministic

Elements used:

Element	Description	In Object	LSD
$ms^*$ (V)	target market share	Firm	TargetMs
$ms_{t-1}$ (V-1)	lagged market share (in the final sector)	Firms	Ms
$s^{MS}$ (P)	smooth parameter for the market share	Market	smoothMs
$Y_{S_{F-s}}$ (V)	Quantity sold by firms in the supplier sector, as inputs for clients	Firm	Sales

$$ms = \begin{cases} ms_{t,S_F} = s^{MS} ms_{t-1} + (1 - s^{MS}) ms_t^* & \text{if sector} = S_F \\ ms_{t,S_{F-s_j}} = \begin{cases} \frac{Y_{t,S_{F-s_j}}}{n_S} & \text{if } \sum_{i=1}^{n_S} Y_{i,t,S_{F-s_j}} > 0 \\ \sum_{i=1} Y_{i,t,S_{F-s_j}} & \\ ms_{t-1} & \text{if } \sum_{i=1}^{n_S} Y_{i,t,S_{F-s_j}} = 0 \end{cases} & \text{if sector} = S_{F-s_j, s \neq 0} \end{cases} \quad (15)$$

As for the demand, the market share of the supplier firms is computed after the computation of quantities produced, and does not determine them.

## 2.6 Quantity produced

For each firm  $i$ , differs by sector  $S$

We assume that firm tend to produce more than the quantity actually demanded (by the market, for firms in final sector) or the pre-orders passed by clients (for firms in the intermediate sectors) in order to maintain some output in stock. We also assume no direct reaction to other firms' production (quasi monopoly model - nor monopoly, nor

perfect competition, as the good produced is not homogeneous, but different qualities for the same product coexist. When the product is homogeneous, in fact, the model falls in perfect competition case) and error and correction of the quantities at each time period, given a continuous process of adjustment to market fluctuations.

### 2.6.1 Quantity adjustment — $q_t^*$ (Q\_null)

Contained in object **Firm** → **Market**

For each firm  $i$  in period  $t$

Equation for the quantity variation to reach the ‘equilibrium’ in the production following the mkt. shifts: stock and quantity changes in the previous periods

Elements used:

Element	Description	In Object	LSD
$POB_{t-1}$ (V)	pre-orders placed by the clients (in the previous period)	Firm	PreOrderBook
$Y_{t-1}^{sF}$ (V)	sales for firms in the final sector	Firms	FinalSales
$sk_{t-1}$ (V-1)	stock production accumulated up to the previous period	Market	Stock
$a^{q^*}$ (P)	parameter that smooths the change in quantity according to the changes in stock	Firm	a_qnull
$b^{q^*}$ (P)	parameter that smooths the change in quantity according to the changes on previous quantity	Firm	b_qnull
$\mu$ (P)	parameter indicating the multiple of needed quantity to be produced to cumulate the required stock	Firm	mu

$$q_t^* = a^{q^*} [\mu (POB_{t-1} + Y_t^{sF}) - sk_{t-1}] + b^{q^*} [POB_{t-1} + Y_t^{sF} - q_{t-1}] \quad (16)$$

### 2.6.2 Target quantity — $\bar{q}_t^*$ (Q\_target)

Contained in object **Firm** → **Market**

For each firm  $i$  in period  $t$

Quantity to produce in the long term equilibrium

Elements used:

Element	Description	In Object	LSD
$a^{\bar{q}_t^*}$ (P)	parameter indicating how much the target quantity follows the quantity changes ( $q_*$ )	Firm	a_qt
$q_t^*$ (V)	quantity variation decision	Firm	Q_null

$$\bar{q}_t^* = \bar{q}_{t-1}^* + a^{\bar{q}_t^*} \cdot q_t^* \quad (17)$$

### 2.6.3 Quantity — $q_t$ (Quantity)

Contained in object **Firm** → **Market**

For each firm  $i$  in period  $t$

Actual quantity produced

Elements used:

Element	Description	In Object	LSD
$a^q$ (P)	parameter indicating how much the actual quantity varies on its own past history and how much on the target quantity (smooth adjustment to target quantity)	Firm	a_q
$\bar{q}_t^*$ (V)	target quantity	Firm	Q_target

$$q_t = a^q \cdot q_{t-1} + (1 - a^q) \cdot \bar{q}_t^* \quad (18)$$

#### 2.6.4 Stocks — $sk$ (Stock)

Contained in object Firm → Market

For each firm  $i$  in period  $t$

Level of production the firm has accumulated in the previous periods, given by the past stock and current production, without the current selling.

Elements used:

Element	Description	In Object	LSD
$sk_{t-1}$ (V-1)	stocks from the previous period	Firm	Stock
$q_t$ (V)	quantity produced	Firm	Quantity
$Y_t$ (V)	quantity sold	Firm	Sales

$$sk = sk_{t-1} + q_t - Y_t \quad (19)$$

## 2.7 Vertical interaction process

### 2.7.1 Supplier selection — $Pr_{i_{S_{F-s_i}}^{i_{S_{F-s_j}}}}$ (Decide)

Contained in object Input → Firm → Market

For each firm  $i$  in a sector  $S$  which produce goods for clients and vice versa.

Probability that a firm  $i$  in the demanding sector  $S_{F-s_i}$  chooses a particular firm  $i$  among the suppliers (in the supplier sector  $S_{F-s_j}$ ). Which is the probability that a buyer uses as input vector  $\vec{w}_k$  the output vector  $\vec{y}$  of a given supplier  $i$  (with its characteristics), thus increasing its demand for production.

Elements used:

Element	Description	In Object	LSD
$I$ (V)	firm's competitiveness index	Firm	IndexQ
$CD_{t=0}$ (P)	Parameter extracted through a uniform distribution on discrete values, which indicate after how many periods buyers choose a supplier: $CD_{t=0} \sim U(t^{min}, t^{max})$	Input	CounterDecision

$$Pr_{i_{S_{F-s_i}}^{i_{S_{F-s_j}}}} = \begin{cases} \Pr \left[ \vec{y}_{i_{S_{F-s_j}}} = \vec{w}_{k, i_{S_{F-s_i}}} \right] & = \frac{1}{\sum_{i=1}^{n_S} i_{S_{F-s_j}}} & \text{if } t = 1 \\ \Pr \left[ \vec{y}_{i_{S_{F-s_j}}} = \vec{w}_{k, i_{S_{F-s_i}}} \right] & \propto I_{i_{S_{F-s_j}}} & \text{if } t > CD_{t=0} \end{cases} \quad (20)$$

### 2.7.2 Selection schedule — $CD_t$ (CounterDecision)

Contained in object **Input** → **Firm** → **Market**

For each firm  $i$  that acquires at least one input from a supplier sector

The function is a simple counter for the decision making. At the beginning for each firm a number between a given interval is drawn (representing a reasonable amount of time for input reserves) ( $CD_{t=0}$  in Eq. 20), while in the following period the function resets to 100. The difference in the supplier choice time, accounts for firms heterogeneity in the production cycle.

Elements used:

Element	Description	In Object	LSD
$CD_{t-1}$ (P)	value of the counter in the previous period	Input	CounterDecision

$$CD_t = \begin{cases} CD_{t-1} - 1 & \text{if } CD_{t-1} \neq 0 \\ 100 & \text{if } CD_{t-1} = 0 \end{cases} \quad (21)$$

## 2.8 Vertical interaction outcomes

### 2.8.1 Order book — $OB_{i_{SF-s_j}}$ (OrderBook)

Contained in object **Firm** → **Market**

For each firm  $i$  in any supplier market<sup>7</sup>

Determines the amount of input ordered to a specific supplier by it(s) buyer(s), which is the input needed for the actual sales of the period. In other words, writes the quantity each supplier has to produce given the sale choice/requirement made in that period by the buyers, i.e. distribute the orders of final good producers to suppliers, after having ‘anticipated’ the orders with *POB* in the preceding period.

Elements used:

Element	Description	In Object	LSD
$q_k^I$ (V)	amount of inputs needed by each firm in the buyer market, for the specific input $k$ bought in that specific supplier market	Input	Tot

$$OB_{i_{SF-s_j}} = \begin{cases} \sum_{k_{SF-s_i} \equiv SF-s_j} \sum_{i=1}^{n_S} q_{k,i_{SF-s_i}}^I & \forall s_i \neq s_j, \text{ if } \vec{y}_{i_{SF-s_j}} = \vec{w}_{k,i_{SF-s_i}} \\ 0 & \text{else} \end{cases} \quad (22)$$

### 2.8.2 Pre-order book — $POB_{t,i_{SF-s_j}}$ (PreOrderBook)

Contained in object **Firm** → **Market**

For each firm  $i$  in any supplier market in period  $t$

Writes the quantity each supplier will have to provide the following period, given the quantity produced in the present one by the buyers (in other words the expected input requirement for buyers, and the expected demand for suppliers); i.e. distribute estimates

<sup>7</sup>It is perfectly conceivable that goods produced in the final market are used by up-stream sectors.

of orders to suppliers for the following period. Here we show the value for the same period in which the actual  $OB$  is computed. In facts,  $POB_t = OB_{t-1}$ . In other words the  $POB$  computed in time  $t$  are equal to the  $OB$  computed in period  $t$ , but refers to the following period  $t+1$ .

Elements used:

Element	Description	In Object	LSD
$q_{t-1}$ (V)	amount of output produced buy the buyer in period $t-1$	Firm	Quantity
$\beta_{k,t-1}$ (P)	production coefficient	Input	quantity_coeff

$$POB_{t,i_{S_{F-s_j}}} = \begin{cases} \sum_{k_{S_{F-s_i}} \equiv S_{F-s_j}} \sum_{i=1}^{n_S} \beta_{t-1,k,i_{S_{F-s_i}}} \cdot q_{t-1,k,i_{S_{F-s_i}}} & \forall s_i \neq s_j, \\ & \text{if } \vec{y}_{t-1,i_{S_{F-s_j}}} = \vec{w}_{t-1,k,i_{S_{F-s_i}}} \\ 0 & \text{else} \end{cases} \quad (23)$$

### 2.8.3 Input quality update — $w_{t,k,h,i_{S_{F-s_i}}}$ (**w**)

Contained in object **InCh**  $\rightarrow$  **Input**  $\rightarrow$  **Firm**  $\rightarrow$  **Market**

For each firm  $i$  that buys from suppliers, and for the the specific inputs  $k$  bought from suppliers (recall the other option is a general input market).

Updates the values of the input qualities  $w_h$  with the values of the output of the supplying firm for the inputs that are bought from the intermediary markets, after having decided the firm that supplies. This process runs before the actual starting of the production (Eq. 1).

Elements used:

Element	Description	In Object	LSD
$y_{t,m}$ (V)	quality features of the output of supplier firm	OutCh	<b>y</b>
$w_{t-1,k,h}$ (P)	the value of the input feature when it is bought in the general input market is static	InCh	<b>w</b>

$$w_{t,k,h,i_{S_{F-s_i}}} = \begin{cases} y_{t,m,i_{S_{F-s_j}}} & \forall k : k_{S_{F-s_j}} \equiv S_{F-s_i}, \forall m = h, \\ & \text{if } \vec{y}_{t-1,i_{S_{F-s_j}}} = \vec{w}_{t-1,k,i_{S_{F-s_i}}} \\ w_{t-1,k,h,i_{S_{F-s_i}}} & \text{else} \end{cases} \quad (24)$$

### 2.8.4 Input price update — $p_{t,k,i_{S_{F-s_i}}}^I$ (**price\_input**)

Contained in object **Input**  $\rightarrow$  **Firm**  $\rightarrow$  **Market**

For each firm  $i$  that buys from suppliers, and for the the specific inputs  $k$  bought from suppliers (recall the other option is a general input market).

Updates the values of the input prices with the values of the output of the supplying firm for the inputs that are bought from the intermediary markets, after having decided

the firm that supplies. This process runs before the actual starting of the production (Eq. 1).

Elements used:

Element	Description	In Object	LSD
$p_t$ (V)	price of the output of supplier firm	Firm	Price
$p_{t-1}^I$ (P)	price of the input feature when it is bought in the general input market is static	InCh	w

$$p_{t,k,i_{S_{F-s_i}}}^I = \begin{cases} p_{t,i_{S_{F-s_j}}} & \forall k : k_{S_{F-s_j}} \equiv S_{F-s_i}, \forall m = h, \text{ if } \vec{y}_{t-1,i_{S_{F-s_j}}} = \vec{w}_{t-1,k,i_{S_{F-s_i}}} \\ p_{t-1,k,i_{S_{F-s_i}}}^I & \text{else} \end{cases} \quad (25)$$

### 3 Short term modifications, more or less defined

#### 3.0.1 Price change

Firms change their price when their  $ms$  has reduced in the previous  $t - \tau$  periods (during which the  $mkp$  is unchanged by assumption), changing their chosen  $mkp$ . It is the only mechanism that allow firms to change strategies in the short period. The mechanism can be inserted in the same period in which they ‘change’ supplier. Modelling could something like:

$$mkp_t = \begin{cases} mkp_{t-\tau} & \text{if } ms_{t-1} \geq ms_{t-\tau} \\ mkp_{t-\tau} - \frac{mkp_{t-\tau} - mkp_{t-1}}{mkp_{t-\tau}} \cdot \alpha^p & \text{else} \end{cases}$$

where  $\alpha^p$  allows the firm to change the mark-up taking into account also the sensitivity of the consumers to the price change, assuming there is perfect information on this parameter.

The other way in which firms can change the price is reducing the production coefficient.

## References

- GALLOUJ, F. and WEINSTEIN, O. (1997), "Innovation in Services", *Research Policy*, **26**, pp. 537–556.
- LANCASTER, K. J. (1966), "A New Approach to Consumer Theory", *Journal of Political Economy*, **74**, pp. 132–157.
- SAVIOTTI, P. P. and METCALFE, S. J. (1984), "A Theoretical Approach to the Construction of Technological Output Indicators", *Research Policy*, **13**, pp. 141–151.