Firms interactions and industrial development: a simulation model*

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Abstract

This paper is part of a broader research that aims at understanding the industrial development (and innovation) processes in less developed countries, and the way in which their different paths affect growth and the capability to compete, focusing on local production (innovation) systems. In the present contribution we draw from this general framework and present a model of production process, with particular attention to the vertical relations among firms. The model represents heterogeneous firms (depicted as complex agents) producing multi-characteristic products. Firms operate in different sectors combining several inputs, possibly bought from firms in other sectors of the model. The results of the simulations suggest different patterns of industrial development related to different initial conditions.

Keywords:

JEL-classification:

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1 Introduction

This paper is part of a broader research that aims at understanding the industrial development (and innovation) processes in less developed countries, and the way in which their different paths affect growth and the capability to compete. For this purpose, we develop a computational prototype for the analysis of Local Innovation Systems (LIS) and the relations between its actors and with external ones (traders, brokers, suppliers, foreign investors, etc.). We argue that both the Network Structure (NS) and the LIS are evolving entities, which have an important role in explaining the industrial development of a defined system (might be a locality or region). We also argue that their initial conditions, the relations with the broader national system, and the way they are aligned with the international networks[1], shape the capability to compete of the whole country (proportionally to their size). *'Competitiveness'* refers not only to the single actors' efficiency or to aggregate macroeconomic indicators, but to the capability of the system to innovate and upgrade, approaching a leading role in production (and innovation). Defining the *LIS*, we refer to the concept of the National Innovation Systems[2] transferred to the local or regional level, and its localised institutions and organisations . We introduce 'a latere' the concept of *NS*, which focuses on the linkages among the local players, in terms of strength, fluidity, extension, flexibility, etc.

In the present contribution we draw from this general framework and focus on the modelling of the production process, with particular attention to the vertical relations among firms. The model represents heterogeneous firms producing multi-characteristic products. Firms operate in different sectors combining several inputs, possibly bought from firms in other sectors of the model. Thus, firms are depicted as complex agents, but not as systems of interactions themselves. The output, following the conceptual framework of Gallouj and Weinstein (1997) consists in a vector of different quality features of a manufactured good resulting from the combination of inputs and firms' own capabilities.

Vertical interactions between buyers and sellers, both local and foreign, are modeled taking into account the different ways in which production relations can be established, and replicated. The features of the final goods depend also on the features of the inputs used, providing an incentive to both users and producers to co-operate to different extents (Lundvall, 1988).

We model the simultaneous decision of the quantity to produce by firms that have input output relations, through mechanism of supplier selection and input orders. Those routines entail procedures on quantity variations, exogenous and endogenous stock changes, and input variation constraints in approaching the target quantity. The dynamic is pulled by consumers' final demand and firms business demand.

The structure of the relations between firms affect the growth of an economic system, depending on the interdependence of the sector inside the economy, and the need to rely on external inputs. Systems dependent only on the external final demand have a limited growth rate with respect to those that develop endogenous production of inputs. The above depends on the technical coefficients of the input, but also on the feedbacks processes that

[1] We borrow the idea of alignment of networks at different levels (local, national and international) as a key variable to investigate and understand 'successful' industrialisation processes from Kim and von Tunzelmann (1998).

[2] Which has been widely defined and analysed, although under different perspectives (e.g. Freeman, 1988; Lundvall, 1988; Nelson, 1993). The central characteristic, which emerges through the various representations, is the view of the industrialisation process as a systematic evolution in which the interaction among the different actors characterises the pattern of each country (also through the interaction with external players).

characterise quality and price features of the exchanged inputs.

We model the process through a dynamic agent based computational model in which variables co-evolve, causalities are defined, and the results portray the relevant emergent properties of interactions. Using Tesfatsion words, "Agent-based computational economics (ACE) is the computational study of economies modelled as evolving systems of autonomous interacting agents" (Tesfatsion, 2001, p.1). While the concepts of dynamic evolving systems and self-organisation have been introduced in economics in the past (although not widely used, unfortunately), the flexibility and power of the computational approach allows for: i) designing heterogenous agents with cognitive (at least endogenous) behaviour[3], ii) self-organisation is an emergent behaviour of the agents that respond to the system, iii) evolutionary dynamics (and selection) derives from agent-agent and agent-system interaction, and iv) the system evolves endogenously, with no external interventions, step by step (Tesfatsion, 2001)[4].

We implemented the model with a language based on a C++, produced by Marco Valente (e.g. Valente, 2002), Laboratory for Simulation Development (LSD)[5], which allows a relatively simple coding and provides a simulation program endowed with a complete set of interfaces to manage any aspect of the simulation.

In the following section we provide a qualitative overview of the entire model and its dynamics. In the third section we describe in more detail the main components of the model, in a qualitative way. The fourth section presents the main outcomes of the model, through simulation results. In particular, we describe how quantities and goods' qualities and prices feedback in economic systems with different vertical relational structures. In section five we extend our consideration to the way in which the different industrial structure affect the economic development, considering the case of external buyers and suppliers. We draw some final considerations in section six. [An appendix provides the analytical description of the simulation model.]

2 Model Overview

This section provides an overview of the model describing the main elements and dynamics in very general terms. The first subsection describes the data structure of the model, indicating the model's labels of the objects representing the entities involved in the simulation. The second subsection describes briefly the main dynamics of the model.

2.1 Data structure

The model represents an economic system composed by several sectors (`Market`) each producing one product defined over a set of characteristics (Fig. 1). Products sold in the same sector are defined over the same set of characteristics and use the same type of inputs, though they can differ about the quality they assume in the characteristics, as well in the qualities of inputs. Sectors record different statistics, like total production, average price, average quality for each of the characteristic (`Characteristic`). In each sector there are several firms (`Firm`), each producing one single product (multi-product firms can

[3] See Wooldridge and Jennings (1995) for a description of agents features in agent based models.

[4] The literature on ACE have been expanding significantly in the last decade and is too vast to be presented in this work. For a general overview, and application in different economic fields refer to Tesfatsion (2002); for another methodological explanation of the agent-based structure, indicated as a 'third way' for social science see Gilbert and Terna (2000).

[5] for more information on the language and the programme refer to http://www.business.auc.dk/~mv/Lsd/lsd.html.

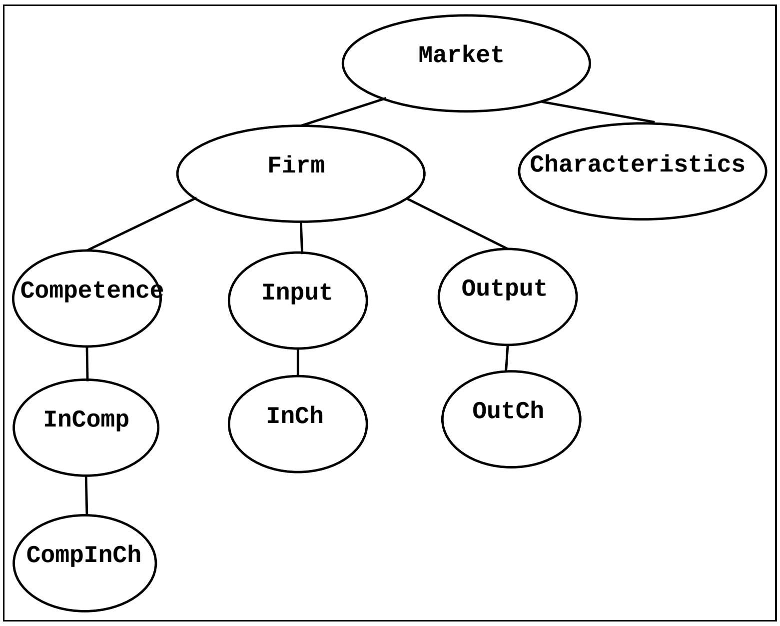


Figure 1: Object structure of the model. The structure is composed by a hierarchical tree. Upper objects contain one or more copies of the lower types of objects, while each lower object is contained in one single copy of an upper objects.

be implemented), whose output is defined both in terms of quantity and in terms of the qualities measured over the product's characteristics (`OutCh`). Firms record financial data (e.g. revenues, profits, etc.) and production data (e.g. stock levels). Firms use production inputs (`Input`) defined over several quality dimensions (`InCh`). Each input is assigned a technical coefficient indicating how many units of that input are necessary for each unit of the firm's output. Firms have different competences in producing each of their own output characteristics (`Competence`). Each of such competences is specific for one input (`InComp`) and one characteristic of such input (`CompInCh`).

2.2 Model Dynamics

The model dynamics represents production processes carried out by firms in different sectors. Firms in a sector may provide inputs to firms in other sectors, use inputs from other sectors, serve a final demand, or any combination of the three. The model can therefore represent linear as well as non-linear production processes. For example, in a non-linear setting a sector i can serve a final demand but also provide input to a sector j that, in turn, provides also one of the inputs to i.

The pace of a simulation run is such that each step represents the time requested to perform small changes in production levels, say the equivalent of a week in an manufacturing industries. Many of the variables therefore are expressed as smoothed changes. For example, a variable X computes at each time t a target, or desired, level X_t^* but its actual value is $X_t = X_{t-1}\alpha + (1-\alpha)X_t^*$, (where $\alpha = (0, 1)$) moving only partially form its current level in the direction of the desired level.

2.2.1 Production

Firms can serve both a (partly endogenous) final demand and the demand from firms in other sectors, called business demand. Whether a firm serves one or both demand types is exogenously determined at sector level. Final demand is computed in two steps: firstly the total amount of the final demand is determined as a function of the average price and average product characteristics at the sectoral level; secondly, individual demand from final consumers is computed as a function of the relative quality of a firm's product in respect to competitors' in the same sector.

Business demand from firms in other sectors is implemented as the direct orders from buying firms to the specific supplier firm (see below for changes in the supplying firms). Firms determine the level of production aiming both at satisfying the expected demand and to maintain stocks at a desired level. Firms firstly determine the desired production levels considering their current final demand (if any), business demand from the previous period (if any) and desired adjustment of stocks. Secondly, they dispatch purchasing orders to their suppliers (if any, sales of exogenous inputs are not recorded but as costs of the purchasing firms) and carry on their production. The purchasing orders are determined by the technical coefficients multiplied by the required production. Subsequently, all the orders are fulfilled, though, in some cases, this may entail undesired stock variations.

During the production process firms determine the quality of their output for each characteristic, as the result of the combinations of their competences and the quality levels of the inputs.

2.2.2 Pricing and Financial Records

The price of a product is determined by the selling firm, and it is undifferentiated for final consumers and client firms. The price is determined using a mark-up over the average variable costs of production. Such costs are determined computing the total costs of production as the amount of inputs times the prices of the inputs. Firms record levels of revenues, profits and financial capital, computed as the profits cumulated through time.

2.2.3 Suppliers' Selection

Firms in the same sector need the same types of inputs. The model admits both exogenous inputs, whose values are assigned in the beginning of the simulation and never modified, and inputs from other sector in the model. For this second class of inputs each firm selects a specific firm in the input sector in the beginning of the simulation run ($t = 0$). Every given number of periods client firms review their suppliers. A new firm from the supplier sector is selected randomly with probabilities proportional to an indicator of the provider 'competitiveness' (quality and price of the input). When a new supplier is selected the client firm replace the old quality and price coefficients of its input with those of the new supplier.

3 Model Description

This section describes the main components of the model, grouping them according to four general aspects of the model. This classification is quite arbitrary, and is meant to be only for simplifying the reading, and provide a better understanding. The first group contains all variables directly concerned with the quantitative aspects of production; the second with the implementation of demand; the third concerns the qualitative aspects of production; the last one on financial book-keeping. Each section actually refers to group of variables and contains a brief description of the implementation.

3.1 Production - Quantitative aspects

The quantity produced by firms is determined in three steps. Firstly, the variation of production in respect to the previous production level, given the expected demand level, is ascertained. Secondly, firms decide the desired production level, under a conservative assumption meant to smooth away volatility of the demand. Thirdly, the actual production level is determined assuming a friction in varying production levels (due for example to non perfect substitutability and variation in the use of inputs — e.g. sudden increase of machinery, or sudden decrease of manpower).

The differences between current production and actual demand are compensated by variation of stocks.

3.2 Demand

There are two types of demand for a firm: firm's final demand and business demand. Total final demand is determined at sector level as a function of the prices and qualities of all products in the market. Firms' individual amount of final demand is derived as their market share times the total final demand. Business demand is the amount of products requested by firms in other sectors. Thus, it is directly determined by orders delivered by client firms to their suppliers. Total business demand is derived by summing up individual firms' business demand.

3.3 Production - Qualitative aspects

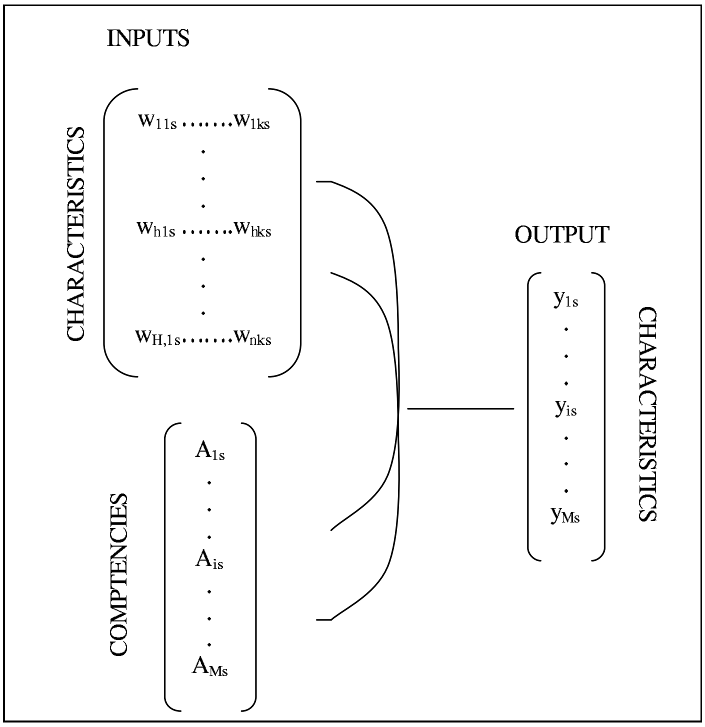
The quality of products sold in a market is defined over an exogenous number of product's characteristics. The quality level of the product for a firm in respect of each characteristic is determined by two factors: quality of the inputs and competences of the firm. The latter represent the capacity of transforming each characteristic of each input in quality levels of the product. The production process (of one unit) takes place by combining inputs and firm-specific competencies, following the framework drawn by Saviotti and Metcalfe (1984), and modified by Gallouj and Weinstein (1997), as depicted in Figure 2.

Each firm thus has a certain amount of initial resources (material and immaterial) and combines them to obtain the final product. We thus adapt the Saviotti-Metcalfe framework depicting a set of $k = \{1, \dots, K\}$ input vectors $\vec{w}_{i,k} = \{w_{1,i,k}, w_{2,i,k}, \dots, w_{h,i,k}\}$ of $h_k = \{1, \dots, H_k\}$ inputs' features[6], which can differ (in both numerosity and value) for each input k used for the production of a good. Similarly, a vector $\vec{\vec{A}}$, of which each element $m = \{1, \dots, M\}$ is a set of K vectors with H_k elements, contains the competencies $a_{m,h_k,k} = \left\{1, \dots, \sum_{k=1}^{K} \sum_{h_k=1}^{H_k} h_k \cdot M\right\}$ required for the use of each specific combination of input and their features, and defines one product feature combining with $\vec{w}_k \times K$.

The price of products is treated in the model as an added characteristic, basically for the fact that it is determined in a different manner: in fact, it is computed as a mark-up over the costs of inputs.

The model can represent the use of either "exogenous" inputs, obtained from sectors not represented in the model (general markets), and of "endogenous" inputs, from sectors represented in the model. This second types of inputs are chosen by client firms every given number of periods, using a random choosing function with probabilities proportional to the average quality of the products in the input sector.

[6] Different inputs can have a different number of characteristics. Inputs are thus represented in the same way of the output



Source: Adapted from Gallouj and Weinstein (1997), p. 544

Figure 2: Qualitative representation of the production process: the combination of inputs (with their own features) and competencies result in a set of output characteristics

3.4 Financial accounts

The model records the financial statutes of firms during their life times. The most important variable is the price of products, determined by firms using a mark-up on the variable costs. Firms record the following financial variables: revenues, profits, financial capital (i.e. cumulated net profits). Further extension of the model may exploit the financial data to implement, for example, investment capacities or a financial sector, currently not implemented.

4 Considerations on the vertical interaction structure and emerging (industrial) development

In the present section we concentrate on the main economic results and intuitions that we draw from some preliminary simulations. In particular, we concentrate on the processes of vertical interactions and how, through specific model parameters and features, it might affect economic industrial development. We highlight the difference between *linear* and *circular* vertical structures of a closed economic system. Hence, we show how the different feedbacks, due to the vertical linkages between firms in complementary sectors, entail quite different economic scenarios.

4.1 Production coefficients, vertical feedbacks and growth of the economy

The input technical coefficients (β_k) enter the model as the *indicators of the actual quantity/ratio of each input used by a firm to produce one unit of output*. Given the decided output quantity described in section 3, firms buy an amount of each input k which satisfy their production needs:

$$q_k^I = \beta_k \cdot q$$

The use of technical production coefficients instead of 'elasticities', or more in general 'contribution to production', is aimed at avoiding discussions on, and forced interpretation of those parameters, which are used when dealing with standard production functions. This approach provides some advantages and some constraints. Among the first, i) we do not have to consider only aggregated inputs[7]; ii) thus, we can attach to each input specific features (as shown in the model descritpion — section 3.3), which represent different ways in which it reflects on the production process and output features; iii) when considering 'virtual' sectors for purely theoretical experiments, we do not need to provide particular explanations on the 'estimation' of elasticities, cross elasticities, residual factors, and the like; iv) and so on... In brief, we can model the production process in a much more flexible way, and with less 'theoretical' constraints, as described above in section 2 [8].

Among the latter, i) the division between sectors is arbitrational, ii) when treating with 'real' sectors it is difficult to acquire information on the actual values of those coefficients through input/output analysis, and iii) the interplay of the β_k's in a high number of related sectors renders the model very sensitive to their values.

Given the way in which we define the coefficients, they can assume any real number. More specifically, we can distinguish between i) coefficients that indicate units (as wheels for a car), ii) coefficients that indicate quantities with different units of measure (e.g. primary resources), and iii) ratio of inputs (e.g. use of a machinery). The first assumes values greater than 1, the second has to be defined, while the latter lower than 1. This in principle seems quite trivial[9], but when considering the relations between sectors of production (input/output), it is less so.

Let us refer to a closed economy producing goods in different sectors, which are vertically connected. When there is a unique direction of input/output relations such that the sectors can be aligned in an ordinal vertical structure from the first provider to the last consumer (i.e. each sector buys inputs only from upper-stream sectors and sell output only to down stream ones), any coefficient value can be consistent, and strongly determines the growth of the economy. Call this the *linear model*. Note that we are assuming the existence of a positive final demand (section 3.2), or the system would be static and no production would ever start. In brief, given the value of the final demand, the bigger are the β_k, the larger is the amount of quantity produced in the economy. To be more revealing, one should consider the value of the produced goods, in order to provide an indicator of growth (including productivity considerations); we leave this aside for the moment, and consider the coefficient a 'rough' explanatory variable for growth.

Things change when, more consistently with the 'real' world, the sectors are intertwined, they all produce an output that is an input for other sectors[10]. Call this the

[7] such as labour, capital and material

[8] You can find an analytical description of the production model equations and implementation in the Appendix A or in Ciarli and Valente (2003)

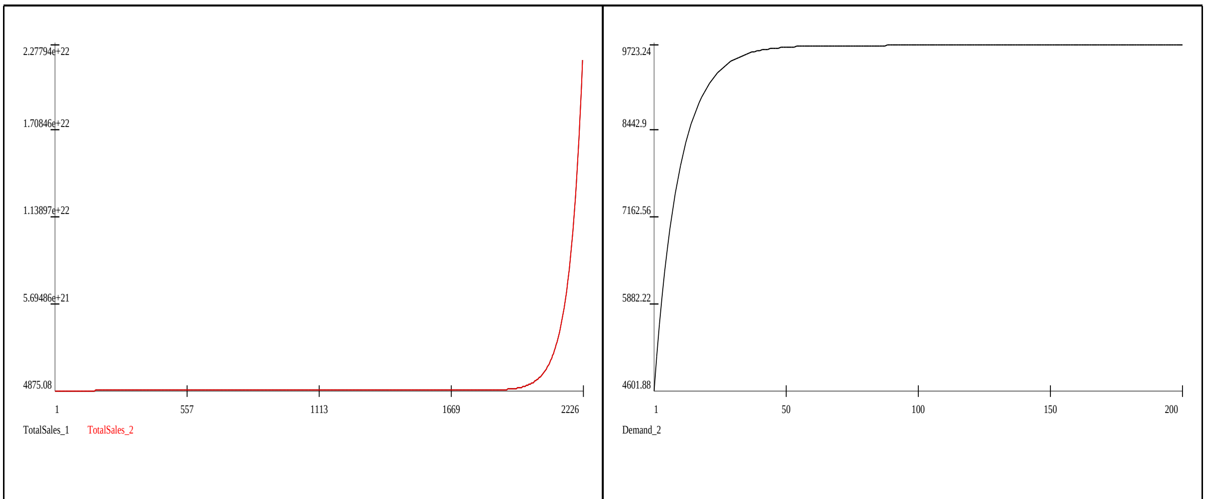
[9] Yes, and probably it is: some of the inputs are divisible, while some are not.

[10] the ordination of the sectors can only be partial and can be done, for example, considering as indicator the number of sectors supplied. Using this criteria, one could think at the primary sector as the 'first' one.

circular model. In this case the values of the β_k can determine the 'explosion' of the economy, and the system overproduces quantities that are not determined by the demand. More precisely, this is the case when the product of the β_k of two interrelated sectors (the output of each one is an input of the other sector (firm)) has a value equal or greater than one. From a mathematical point of view we have an exploding series, and from an economic point of view, it simply makes no sense. The system is *out of production equilibrium*.

4.1.1 Simulation results

In Figure 3 we represent the value of the total sales of two sectors, each one composed of one single firm: the second sector (2) sells its product to the final market and to the first sector (1) (which use it as an input), which conversely sells its output to sector 2, which use it as an input. Hence, firms in both sectors use inputs produced by the firms to which they also sell their output. If we compare total sales (a) wuth final demand (b), we clearly observe that the latter is much lower than the output sold (which tend to infinite). This is the case because the two sectors, in order to satisfy the final demand, keep increasing the quantity of input required one from the other.



(a): Total sales of both sectors (b): Demand for final sector

On the x-axis time and on the y-axis the values of the variables

Figure 3: 'Exploding' economy and the role of β_k coefficients in a vertically intertwined economy. Total sale and final demand of a model with two sectors, each with only one firm and cross production of the coefficients equal to one ($\beta_2^{sF} \cdot \beta_2^{sF-1} = 1$).

In general, the technical coefficients of two interrelated sectors need to be either both less then 1, or one the inverse of the other, such that their product is lower than one. In the two-sector-two-firms case this is quite straightforward and can be easily shown with the following production system:

$$\begin{cases} q^1 = \beta_k^2 \cdot q^2 + D \\ q^2 = \beta_k^1 \cdot q^1 \end{cases}$$

where q is the quantity produced, the pedix refers to the input and the apex numbers to the sector. Hence, after two simple algebraic passages the solution for the system is:

$$\begin{cases} q^1 = \frac{D}{1-\beta_k^2 \cdot \beta_k^1} \\ q^2 = \beta_k^1 \cdot \frac{D}{1-\beta_k^2 \cdot \beta_k^1} \end{cases}$$

First of all we note that, as we have assumed until now, a final demand must exist to start a production process[11], even if the sectors are totally interrelated. Secondly, we derive the mathematical conditions to reach the eventual equilibrium (*ceteris paribus*). Nonetheless, with the simple mathematical solution we can only conclude that the quantity produced is infinite if $\beta_k^2 \cdot \beta_k^1 = 1$ and negative (!!) if $\beta_k^2 \cdot \beta_k^1 > 1$. Conversely, with the simulation tools we are able to show the deviation from the equilibrium and the pattern of production explosion (Figure 3)[12]. The interpretation of the result at the end is quite similar (quantity diverges, or the system is not sustainable) but in the second case we are able to explain it observing the *dynamic* of input quantities and the relation with output. The case for simulation is particularly important, as in our model the conditions are complicated by the overall complexity of the system and the endogenisation of variables, with an increase in the system arguments (e.g adding firms, quantity rules, markets, coefficient variation, and the like).

When we refer to a *production equilibrium*, we do not consider it a unique path, and we refer to a closed economic system. Thus, concerning the sole production system, we might better refer to its *consistency*, or *existence*. I.e. the conditions under which each sector can provide sufficient inputs to the others without having to acquire more inputs from them than they would have to use for their own production, and so on. When both sectors i and j need more than one unit of inputs to produce their output, they simply cannot produce it, the system is no more sustainable. Hence, the system as a whole has a maximum production level it can achieve, given by the coefficient constraints. Moreover, the 'global' economic system then depends on the dynamics of the demand, competencies, production processes — change in the coefficients along time, technology, change of the inputs, etc. Hence, we can only reach a conclusion upon the existence of a production constraint (an asymptotic equilibrium), which reflects on an economic constraint, *ceteris paribus*, observing the interaction of micro agents. We rely on the emergent property of the constructed system to draw our conclusions, we do not assume equilibrium or constraint existence from the beginning of the model[13].

In Figure 4 we show the dynamics of a system with two sectors, each with one firm, with the same parameter initialisation of the above Figure 3, but with lower production coefficients (both lower than 1).

First of all the system does not explode, and the production reacts to the final demand requirements. Secondly, it is possible to observe that the production system reaches its long run equilibrium only after many periods, due to the stickiness in the adjustment process[14] (in particular in this simulation it is still not reached after 2226 periods). Any

[11] Quite trivial again.

[12] When the product of the coefficients is bigger than one, the quantity produced explodes and is not negative

[13] In this respect we agree with Leigh Tesfation in her intervention in the discussion on the use of equilibrium concept in economic theory: we can use the equilibrium concept and build theory on it, if we show it is the emergent property of a self organising system, but not build an economic theory (and micro behaviour) on the assumption of the existence of an equilibrium (round table at the Wild@Ace conference, Turin October 4^{th})

[14] For reasons of space we do not present comparisons among different simulation with different parameters' values, but it is possible to show that i) when the parameter of adjustment of the quantity variation with respect to previous stocks (see section 3.1) tend to 0 the cycles amplitude tend to 0; ii) when the para-

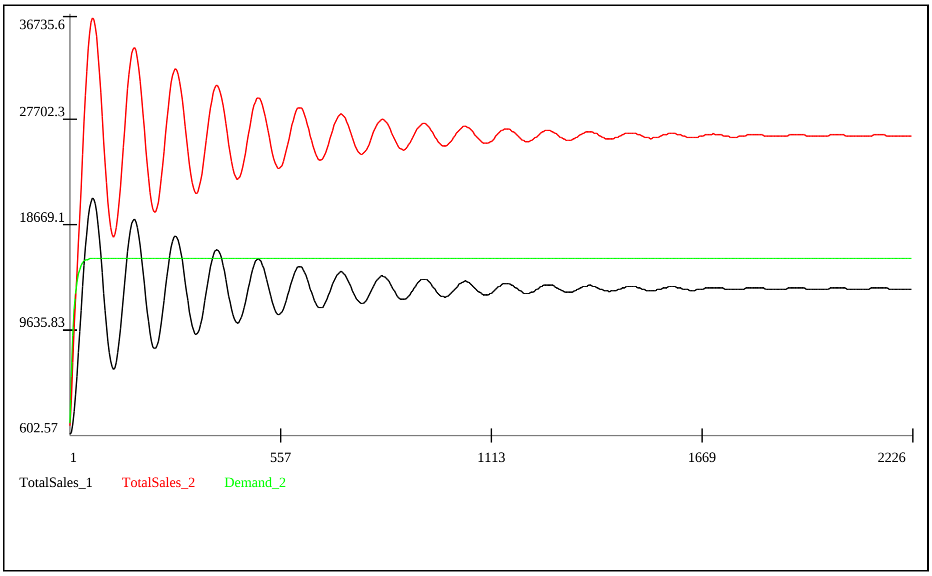


Figure 4: Total sales and final demand of two intertwined firms in two differnete sectors, with β_k cross product lower than one ($\beta_2^{sF} \cdot \beta_2^{sF-1} = 0.4$). The firm in the final sector satisfies both the final demand and its input supplier with its output.

change along the market evolution in the demand, input features, coefficient or anything else, would clearly change the long run state (and start new adjustment oscillations).

Moreover, we are representing a very simple economy (two agents), or an economy with representative agents, which inherently has a very simple dynamic. Adding a higher number o firms in the market causes a neverending oscillation around the long term status.

Things do not change if we consider an open economy, with the possibility of acquiring inputs from an external system (e.g assuming a world composed by two economic systems). Better said, they would change for one of the economies, but not for the 'global' system which comprises both of them. In the circular model, with non stable coefficients, the sector that needs more inputs than its provider sector can produce, would have to rely on the external system of suppliers. We can think of any number of economic systems levels, and easily reach the global production system, in which the possibility of acquiring from external systems expires. This has straightforward implications for primary resources producers, both in terms of production rate and value added.

4.2 Other feedbacks: prices and quality features

While the production coefficient shows important feedbacks on the quantity produced, the vertical relations between firms affect the system through other two important variables of the economy: prices and qualities of the goods. Both of them cause changes in the demand (final and business) and in the development pattern of the production system, depending on the way firms are interrelated. In facts, as described in section 2.2.3 the output features enter linearly as inputs for the buyer firm.

meter of adjustment of the quantity variation with respect to previous quantity production tend to 0, the production oscillates continuously; and iii) when the parameter of adjustment toward the actual quantity (with respect to target) goes to 1, the equilibrium is reached much faster, with smoother oscillation, while when it goes to 0 the oscillation converts into picks.

4.2.1 Prices

The output price is given by a mark-up decision of the firm on the variable costs (section 3.3). The variable costs, through the input prices, are determined also by the production coefficients. This is again quite intuitive: the coefficient provides information on the contribution of the price of each input to the final price. What is slightly less intuitive is that the 'circular' model shows a self reinforcing dynamics (Figure 5), providing some indication on the role of linkages in both a closed and open economy.

4.2.2 Qualities

Given the competencies internal to each firm (which have different dynamics that have not been considered in this paper), qualities of the input directly enters in the input vector of buyer firms (section 3.3). When all the sectors are intertwined, also the qualities follow a recursive dynamic, although to a lower extent given the absence of the multiplier. An economy in which the firms are strongly integrated, the quality features of the final good increase more than where there is a linear vertical interaction.

4.2.3 Simulation results

In the following Figure 5 we compare the average quality values ($\overline{y}_i$ in figure 2)[15] and average prices across sectors for both a linear and a circular productive system. In both model initialisation the economy is formed by four sectors, among which the last one uses the inputs of all the upstream sectors and sells only to the final market. Conversely, the intermediate sectors use only two inputs. The remaining parameters are identical in the two cases, including competencies and input coefficients. The two economic systems differ in their linkage structure as following: in the *linear* model,

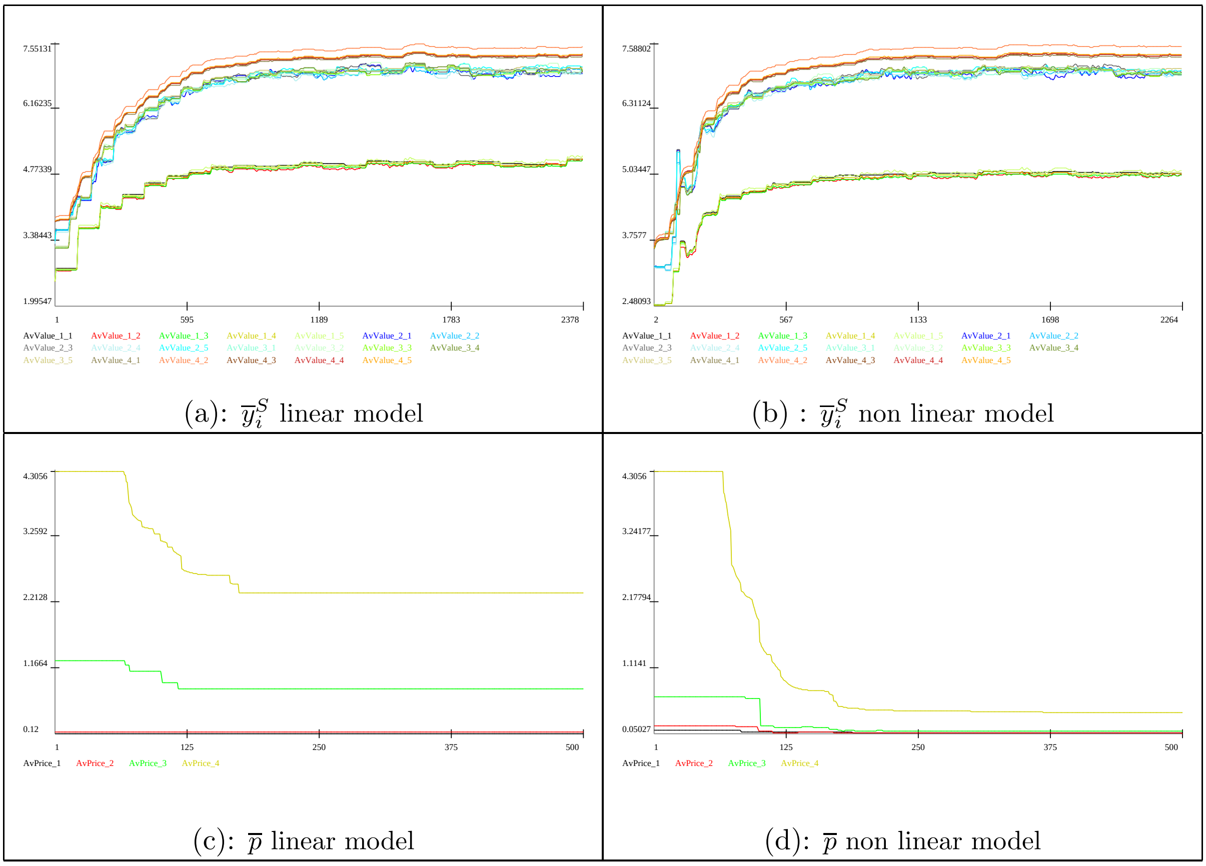
- the first sector buys both inputs from a general input markets (external to the economy);
- both remaining intermediate sectors buy one of the inputs from the general input market, and
- the second sector buys the remaining input from the first sector, the third from the second;

in the *circular* model,

- the first sector buys one input from the general market and one from the second sector
- the second sector buys one input from the first and one from the third ($\beta_k = .05$ and .6 respectively)
- the third sector buys one input from the first sector and one from the second (both $\beta_k = .6$)

The upper boxes of Figure 5 ((a) and (b)) show the average quality of the output produced by the different firms in the different markets. In particular, the higher group of

[15] The average quality value for a firm is a simple average of the value reached by each feature, given by the competencies and inputs used.



(a): $\overline{y}_i^S$ linear model (b) : $\overline{y}_i^S$ non linear model

(c): $\overline{p}$ linear model (d): $\overline{p}$ non linear model

Figure 5: Comparison of the average values and prices in a four sector economy between a fully interconnected production system and a linearly interconnected one. The higher groups of series of average values are the ones of the intermediate sectors, while the lower represents the final sector

series show the dynamic for the firms in the final market, while the lower group represents the series for the firms in the first market. The disparities between the linear and circular models in the intermediate sectors are not that evident. Nonetheless, in all the markets firms, on average, reach better qualities with the non linear model, both when considering the average value through time and the maximum value reached. Moreover, those small differences strongly reflect on the firms' index of competitiveness and on their final demand, as discussed in the following section 5.1.

Not surprisingly, the disparity is stronger when considering the final sector: output quality is higher when final buyers acquire from an interconnected production system than when they buy from a linear system. This preliminary result shows that the overall interactions in the system produce quite different emergent properties when concerning the different structures. As a consequence the supplier firms represented in (b) are likely to become more 'competitive', given they are providers of a better fitted buyer. The constantly lower value in the first sector depends on the fact that they acquire inputs on a general market in which the quality features do not change through time, given the absence of feedbacks. Similarly, the higher value for firms in the final market is due to the fact that they acquire three inputs from suppliers which improve qualities through time.

A second relevant result is the speed of transmission of the qualities from input to output. In a well interconnected system of production, in which the manufacturing of each sector marginally depends on the other producers present in the economy, the average values of quality increase much faster. In fact, it is well evident from Figure 5 that the series' slope near the origin is higher in box (b).

If, on the other side, we consider the dynamics toward the temporary equilibrium of the prices (boxes (c) and (d)), we find an even larger difference in the final outcome of the production chain due to the role of the multiplier[16]. Again, in the non linear model the price change percolates much faster in the system, and firms clearly acquire a a price advantage[17].

5 Local depth and external buyers

The previous section provides various elements for the evaluation of different patterns of industrial development and the relation with the interconnectedness of the system, in terms of distribution and percolation of prices, qualities and quantities. In the present section we comment on the aggregated results of the economy, which embed the above dynamics and allow for further considerations and the proposition of further intuitions to explore. We remind that we are not providing in this paper robust results on the development processes, as we are not deeply analysing the parameter space. We are more intentioned to propose (for discussion) some possible explanations of development and growth phenomena, through the understanding of the cause-effect relations behind the variables. Given the deliberate high flexibility of the model, it would be possible to fit it in to a high number of different conditions, but we are here more interested in its general behaviour. Thus, we fix the parameters which are not particularly interesting at this stage (or we do not want them to be so as they would render the overall picture more complicated) to a unit value, and use 'sensible' values for the interesting parameters[18].

In particular, we concentrate the discussion around the 'depth' of the economics system, and the related argument of the role of external relations with buyers and suppliers.

5.1 Feedbacks, economic depth and structure

Recalling the comparison between linear and interconnected productive systems, we have seen how in the first case the quantity produced by each sector strictly depends on the coefficients of the downstream ones, while in the second case the relation is much more complex, and the 'multiplier' of the economy is definitely higher (*ceteris paribus*). We introduce the concept of 'economic depth', or sectoral depth, simply referring to the number of sectors that are present in one economic system, given the total number of sectors in the 'global' economy. Thus, *a country has a deep economic system if it accounts for a given (high) number of sectors, which have input/output relations.* Our simulation results show that, in general, as expected, a deeper economic system produces more than a one sector economy (e.g. compare the total sales in the final market — TotalSales_4 — and the total sales in intermediate markets in Figure 7,a: the former sells only to the final consumers, while the latter satisfy the demand of different (two) buying sectors — TotalSales_1).

Still, one could argue that if a country is strongly specialised in one good and produces it with the best quality and the lower prices, it can gain market shares, and hence increase its production independently from its economic depth. Said in a different way, one could argue that there is a trade off between the specialisation, which through Learning By Doing

[16] We recall that in our model both qualities and prices are the determinants of the demand.

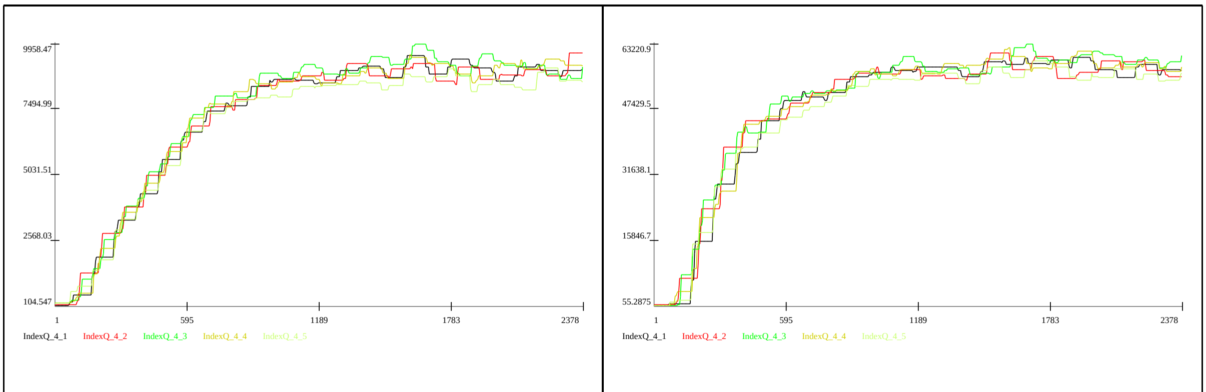
[17] In interpreting those results we should consider that in the simulation of the circular case also the different input coefficients, which are lower than 1, play a role in reducing the prices.

[18] For a more comprehensive discussion on the methodological approach we propose and adopt in this series of works refer to Ciarli and Valente (2003).

(LBD) increases the competencies, hence the final demand through quality increase, and the sectoral differentiation. Nonetheless, in this preliminary work we are considering only ‘standard’ variables of production (prices, qualities and quantities), without entering in the domain of knowledge, competencies, capabilities and the like, for which the model is disposed, but which require further and different analysis. For example, the above statement on LBD is valid if we assume that there is a high knowledge diffusion in a given productive system, through firms at the horizontal level. While we acknowledge the role of horizontal relations, it is one of the scope for further analysis through the model extension: in fact, it should be analysed as an emergent property of agents behaviour and not assumed as a given one. Moreover, even when assuming a positive role of LBD at the sectoral level, we have shown that competencies represent only one side of the production, being input quality features the other. If, in other terms, the economy considered is not able to acquire sufficiently good inputs, the features of its final good will still be relatively low. This is the case, when the productive system is ‘flat’[19].

The role of depth becomes significant in the case of interrelation between sectors. We thus focus the definition of *economic depth* on those systems that have a high number of *interrelated sector, in a non linear way*. When this is the case, an increase in output of one sector increases the output of the remaining ones (and of its own one if producing a good which is also an input). The economic effect is not limited to quantities but also to prices and qualities. Thus, for given coefficients of production with values lower than 1, the index of competitiveness of the final sector is much higher when the economy is characterised by a high number of intertwined sectors, as shown in Figure 6,b.

As a preliminary interpretation one might think again at the specialisation of the economy and its well known advantages, in a Smithian fashion: the single production processes become more efficient and increase the fitness of the final product, exactly as in the pin factory. Nonetheless, this first interpretation is not complete. In fact, we argue that the sectoral specialisation has to characterise each single economic systems. The growth effect we find in our simulations has more to do with the direct interactions inside the system, and the fast percolation of prices, qualities and quantities that derive from the feedback processes, as we have underlined above (section 4).



(a): Competitiveness index linear model (b) : Competitiveness index non linear model

Figure 6: Comparison between linear and non linear production systems: disaggregated indicators. Indexes of firm competitiveness in the last of a four sector economy, which sells to the final market.

[19] One could than argue that, assuming sectoral LBD, if all the economies specialise in one production they will learn to produce in a better way each input. But following with those consideration would require the introduction of many other factors that for the moment we prefer to live on a side and consider constant (i.e. balance of payments, terms of trade, relations between users and producer, etc.).

Moreover, given the better fitness of the firms in the final market, the increase in their market shares in the overall economy again feedbacks on the growth of the considered system. In figure 6 we show the differences between a linear (a) and a non linear model (b) and the second one clearly has higher emergent values of firm competitiveness indicator.

Thus, a second interpretation would drive us toward well known Import Substitution (IS), or Infant Industry (II) strategies, controlling for the role of knowledge transfer, which is assumed fixed in this model. The second caveat regards the role of final demand, which we have seen is crucial in driving the single economic system. In facts, by model construction, those results are due more to the differences in output qualities, rather than prices[20].

Thirdly, if we interpret the non linearity as a proxy of link strength between the actors of the system (in a vertical — input/output — direction), economies with strong and pervasive linkages achieve better growth and development conditions[21].

Finally, there seem to be a tread off between specialisation effects and economic depth.

5.2 The role of external relations

If we relate the argument to the industrial structure in developing countries' economic systems, in which sectors of the industry imports most of their 'high-tech' inputs, we would be in the condition in which the low depth of the local economy reduce the extent of the production (as in Figure 7,a). Considering, for example, the well known experience of the maquila industry, which partially drives (or have driven) the Mexican economy, we basically have only one producing sector, which assemble single parts of imported goods and have scarce local relations. Dutrnit and Vera-Cruz (2002) argues that, so called 'third generation' investment have increased their technological content, but not their relations with local suppliers and economic system. Bair and Gereffi (2001) argue that the new maquila investments have actually displaced the local first-tier suppliers, while according to Dussel (1999) the North American investors control the whole production. Things are not different when changing sector or countries, as clearly stated in the following example by Cimoli and Katz (2001, p.1):

> Launching the Ford Taunus to the Argentine market, back in 1974, demanded some 300 thousand hours of domestic engineering efforts carried out by a local team of 120 professionals employed by Fords Engineering Department. These people were responsible for generating a steady flow of incremental units of production organization and engineering knowledge required for the adaptation of the German-designed blue prints to the local environment, to the available raw materials, to the idiosyncrasies of the Argentine plant not bigger than 10% of Fords production facilities in Germany to the technological capabilities of domestic subcontractors producing parts and components for the referred vehicle, and so forth. One and a half year of domestic engineering activities were required in order to introduce changes and adaptations in the design of the vehicle, in production planning and organization routines, in

[20] Even if in absolute terms price differences between the circular and linear model are much higher than average quality differences (see Fig. 5, by construction the final demand is definitely more sensible to differences in the latter.

[21] From the above results one might conclude that, in the limit, an autarchic system is the first best option. Yet, we acknowledge the positive role of specialisation in both learning (increasing firm competencies) and productivity (reducing input coefficients), quite strongly supported by theoretical and empirical evidence. Thus we would need to set a limit to sectoral differentiation. Moreover, as already mentioned, the final demand plays an important role in guaranteeing the growth.

> the technologies employed by local subcontractors for the production of parts and components, etc before the car could be brought to the market. Domestic content for such car was close to 90% of the total value of the vehicle.
>
> [...]
>
> The production organization model and the technological regime associated to the production of the Ford Taunus in Argentina - which describe at its best the functioning of this industry and its localized technological regime in the above mentioned country in the 1970s and 1980s no longer exist to-day. Ford Argentina is now part of a worldwide integrated production system , working on-line with other Ford subsidiaries around the world, assembling with less than 50% local content, and with no domestic engineering efforts whatsoever a world-car which has become something of a commoditiy. Though the productivity gap between the Argentine automobile industry and the worlds frontier can be shown to be somewhat smaller than in the past, Argentinas vehicle industry is still far behind international standards. It currently exports close to a quarter million vehicles per annum to Brazil under Mercosur preferential conditions but it could not capture new markets in recent years even in spite of the fact that it is currently operating at less than 60% of installed capacity. Moreover, it has now become more of an assembly operation of imported parts and components with little, if any, incidence upon the development of domestic technological capabilities and mechanical engineering skills. Its well-established synergies and externalities vis a vis the local metalworking industry no longer can be argued to be significant.

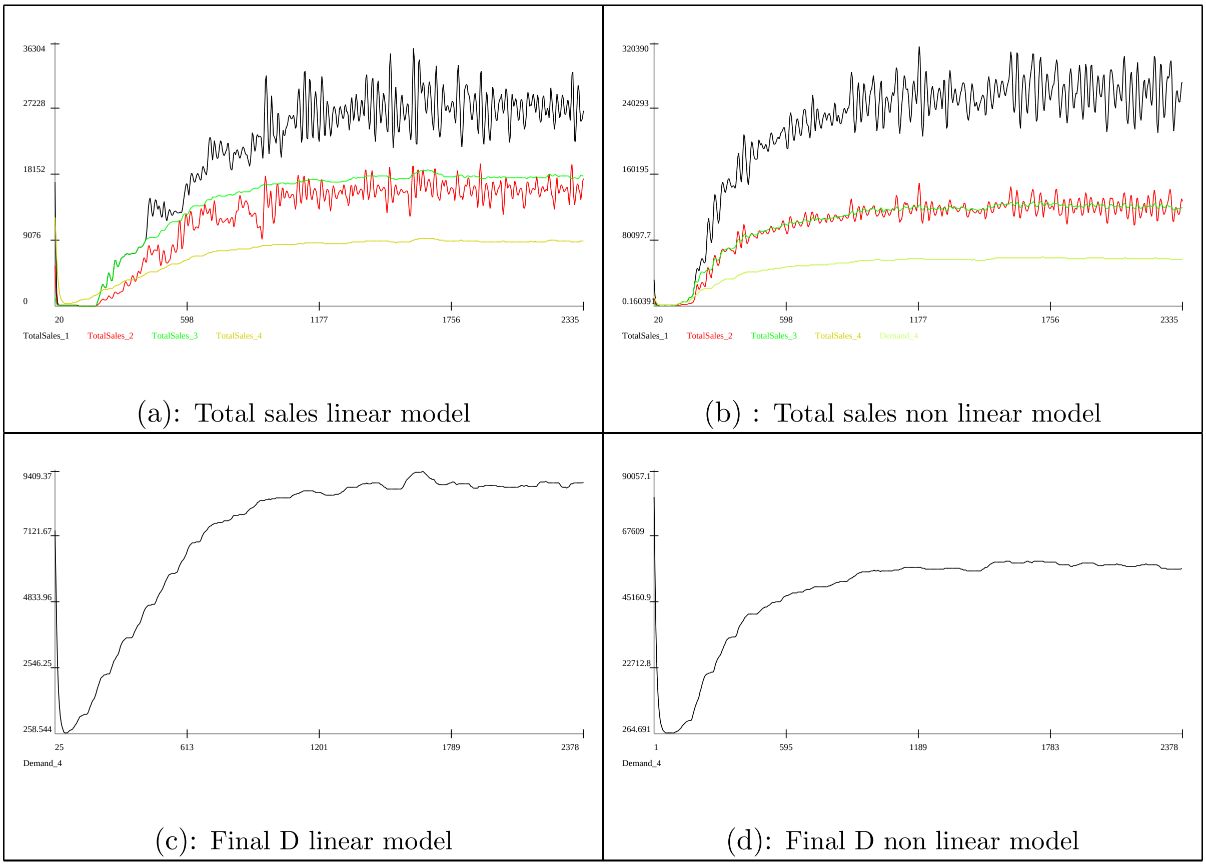
Similarly, countries which base their development only on primary resources, are shown to struggle in the international competition, and reduce their rates of growth (e.g. Cimoli and DiMaio, 2002), when not compromising their process of industrial development (e.g. Katz, 2001).

On one side the output depends only on the final demand from the market, which when the system behind is not well interconnected is lower (due to the mechanisms discussed in section 5.1), as shown in Figure 7,c. The lower demand impact its lower value on the production of the down-stream sectors in the interested economic system (country). On the other side, the local economy does not grow (develop), given the low feedbacks process (low linkage), using a Hirschmanian perspective (e.g. Hirschman, 1977). Clearly the two processes interact and self-cumulate.

6 Final remarks

We have proposed a model representing the production processes of heterogeneous firms in different sectors, with multiple possibilities of vertical interaction. The basic units of the models are firms, assumed to be single-product[22] producers. Firms's definition include: competencies to be applied on the different characteristics of each input, production coefficients, prices and financial accounting. The model assemble the different firms in different sectors representing the dynamics intermediate and final production of firms triggered by the final demand, itself endogenously determined by the quality of final products. Firms quantitative decisions are based on orders from customers, which, in turn, can be revised during the time between orders and actual purchase. Differences between production and sales are matched by variations of stocks, that firms try to maintain at a “normal” when

[22] Though focusing on markets for products, the model is also able to represent markets for services.



(a): Total sales linear model

(b) : Total sales non linear model

(c): Final D linear model

(d): Final D non linear model

Figure 7: Comparison between linear and non linear production systems: aggregated indicators. Sector's total sales in a four sectors economy, and demand for the final sector

deciding their production plans. This aspect of the model is an extremely flexible tool in order to represent a wide variety of production processes, where, for example, all sectors use products by any other sector, besides exogenous inputs.

The applications of the model discussed above focus on the levels of production quantities under different technical coefficients, i.e. amounts of inputs for each unit produced. We have been able to study the results stemming from stream-like production processes versus circular ones. In the second case, we observe endogenous growth caused by the required intermediate product. Different parameterization determine several typologies of this growth mechanism, ranging from explosive growth, cycles or simple equilibrium convergence. Such different results expose the differences of development patterns between "flat" production systems as opposed to "deep" ones. In the former case, final demand, possibly from foreign sources, is the only driving force of growth, making the system extremely dependent on external factors. In the second case, external factors play only the role of triggers of an endogenous dynamics, potentially allowing the emergence of a self-sustainable economic system.

The methodology adopted, based on computer simulations, does not limit the analysis to the study of the final data sets produced. This would have left us with only the possibility to compare simulated data with real ones, wondering how both kinds of data have been produced. Rather, we have been able to dig into the simulated world in order to follow each an every element of the model at any moment. Any simulated events have been traced to the elements of the model that caused them in the first place. As a result, we have obtained a set of robust causal links rooted in the logic of the simulated model, that allow a sensible interpretation of the real world events, independently from the actual similarity of the simulated numerical values with the real ones.

For example, under the "explosive" scenario the model shows the need of infinite amounts of some inputs necessary in order to provide a given amount of final demand. Such result, though clearly absurd if judged with the yardstick of realism, suggests interesting considerations. For such a system to be economically viable it is necessary to organize the technology such that the bottleneck is formed by the most largely available input, and to keep its price down. In the international trade system this seems to be the role of raw materials, like oil, but also copper, coffee beans etc. Such inputs, in effect, receive a constantly increasing demand with falling prices, although they are the crucial components in many relevant markets.

The model allows a much more ambitious analysis than those presented here. We have not discussed, for example, the role of technological innovation and diffusion, which would entail the endogenisation of the competencies' parameters. This and other aspects can be added to this model in order to study their effects alongside the aspects already considered, following a gradual extension of the analysis.

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A Analytical model description

The model is implemented in Laboratory for Simulation Development - Lsd[23]. Such language implements a simulation step as the updating at the generic time t of all the variables contained in the model. In the following paragraphs we describe with the minimal formalism the main variables used in the model explaining the computation used to update their values at the generic time t.

To simplify the reading we will omit the time index unless it expresses lagged values, in which case we will use the usual notation X_{t-1}. We will also omit, in general, the indexes for the different instances of the variables. For example, each firm in each sector contains many instances of variables for the market share but we will not use indexes for firm, sector and characteristic. The computations make use, in general, of values from other variables and parameters of the model. Listing these elements we will specify the objects in which an element is contained. Unless otherwise specified the reader must understand elements in the same type of objects as being used from the same instance. When this is not the case and index, and its explanation, will be provided.

The paragraph for a variable is composed by the following sections (comments are in small types):

Variable Title— X (`VarX`)

The title of the paragraph reports: extended name of the variable, its symbol used in the text and the label in the model's implementation.

Variable representing behaviour ... of the entity ... computed as a function ...

Text commenting the variable meaning and verbal description of its equation

Containing object: `Son` → `Father`→ `Gran-father` Object containing the variable and its ancestors

Elements used: Table listing the elements used in the equation

Element	Description	`In Object`	`LSD`
Y (V)	descr. of Y	`Son`	`VarY`
Z (V-1)	descr. of Z	`Father`	`VarZ`
α (V) Symbol and type: (V)=variable, (V-1)=lagged variable, (P)=parameter	descr. of α extended name of the element	`Grand-father` Obj containing the element	`alpha` Label used in the code

$$X = f\left(Y, Z_{t-1}, \alpha\right)$$

Equation's expression. Note that there are no indexes, meaning that the Y used when computing the generic X must be the instance contained in the same object, while the Z will be contained in the ancestor object.

A.1 Production - Quantitative aspects

The quantity produced by firms is determined in three steps. Firstly, it is determined the variation of production in respect of the previous production level, given the expected

[23] See `www.business.auc.dk/lsd` for further details on Lsd.

demand level. Secondly, it is determined the desired production level, under a conservative assumption meant to smooth away volatility in the demand. Thirdly, the actual production level is determined assuming a friction in varying production levels.

Differences between current production and actual demand are compensated by variation of stocks.

A.1.1 Quantity adjustment — q_t^* (`Q_null`)

Desired variation of production in respect of previous period's production. The variation is determined by two goals: producing the same amount of quantity actually sold (both to business and final demand); and producing to keep stocks at the desired level.

Note that the model can also implement "non-linear" production processes, where a sector is both a client and a supplier of another sector. For this reason, the implementation of the model is such that all firms in all sectors make their production decisions in parallel. Consequently, no firm knows with certainty the actual demand it will receive by other firms. Firms use past orders by client firms as an estimate of current business demand.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
OB_{t-1} (V)	Orders placed by clients in the previous period, used as an estimate of the current period orders.	`Firm`	`PreOrderBook`
$Y_{t-1}^{s_F}$ (V)	sales for firms in the final sector	`Firms`	`FinalSales`
sk_{t-1} (V-1)	stock production accumulated up to the previous period	`Market`	`Stock`
a^{q^*} (P)	parameter that smooths the change in quantity according to the changes in stock	`Firm`	`a_qnull`
b^{q^*} (P)	parameter that smooths the change in quantity according to the changes on previous quantity	`Firm`	`b_qnull`
μ (P)	parameter indicating the multiple of needed quantity to be produced to cumulate the required stock	`Firm`	`mu`

$$q_t^* = a^{q^*}\left[\mu\left(POB_{t-1} + Y_t^{s_F}\right) - sk_{t-1}\right] + b^{q^*}\left[POB_{t-1} + Y_t^{s_F} - q_{t-1}\right] \tag{1}$$

A.1.2 Target quantity — $\overline{q}_t^*$ (`Q_target`)

Containing object: `Firm` → `Market`

Desired level of production, computed as a smoothed value between the previous period desired production and the new desired variation.
Containing object: `Firm` → `Market` Elements used:

Element	Description	In Object	LSD
$a^{\overline{q}_t^*}$ (P)	parameter indicating how much the target quantity follows the quantity changes (q_*)	`Firm`	`a_qt`
q_t^* (V)	quantity variation decision	`Firm`	`Q_null`

$$\overline{q}_t^* = \overline{q}_{t-1}^* + a^{\overline{q}^*} \cdot q_t^* \tag{2}$$

A.1.3 Quantity — q_t (Quantity)

Actual quantity produced, computed as a smoothed variation between the previous period's production and the new target production.
Containing object: Firm → Market
Elements used:

Element	Description	In Object	LSD
a^q (P)	parameter indicating how much the actual quantity varies on its own past history and how much on the target quantity (smooth adjustment to target quantity)	Firm	a_q
$\overline{q}^*_t$ (V)	target quantity	Firm	Q_target

$$q_t = a^q \cdot q_{t-1} + (1 - a^q) \cdot \overline{q}^*_t \tag{3}$$

A.1.4 Stocks — sk (Stock)

Stock level, computed as the previous period's stock, plus production and minus sales.
Containing object: Firm → Market
Elements used:

Element	Description	In Object	LSD
sk_{t-1} (V-1)	stocks from the previous period	Firm	Stock
q_t (V)	quantity produced	Firm	Quantity
Y_t (V)	quantity sold	Firm	Sales

$$sk = sk_{t-1} + q_t - Y_t \tag{4}$$

A.2 Demand

There are two types of demand for a firm: firm's final demand and business demand. Total final demand is determined at sector level as a function of the prices and qualities of all products in the market. Firms' individual amount of final demand is derived as their market share times the total final demand. Business demand is the amount of products requested by firms in other sectors. Business demand is determined by orders delivered by client firms to their suppliers. Total business demand is derived by summing up individual firms' business demand.

A.2.1 Total Sales — Y (Sales)

The total sales of a firm is the sum of its individual business demand and final demand.
Containing object: Firm → Market
Elements used:

Element	Description	In Object	LSD
OB (V)	Order book, i.e. the quantity requested by client firms	Firm	OrderBook
Y^{S_F} (V)	sales in the final market	Firm	FinalSales

$$Y = OB + Y^{S_F} \tag{5}$$

A.2.2 Business Demand — OB (`OrderBook` and `PreOrderBook` = `OrderBook`$_{t-1}$)

Business demand for a firm. This variable is computed after every firm in every sector updated its production variable. Given the technical coefficients firms then send purchasing orders to their suppliers in the up-stream sectors. The order book of a firm sums up all the orders received by client firms, i.e. firms to which the firm is a supplier.

The model implements firms producing one single product, so that there is no need to specify the sector if one indicates a specific firm. Obviously, firms selling only in final markets have this variable constantly equal to zero.

Note that his variable is used for two different purposes. Firstly, its lagged value is used as a proxy to estimate the current business demand, and determine the production decision of the firm. Secondly, its current value, as the actual business demand, determines the level of sales.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
qI_i^j (V)	Amount of input requested by firm j supplied by firm i.	`Input`	`Tot`
$Client\{i\}$	Set of firms buying products from firm i	- - -	- - -

$$OB_i = \sum_{j \in Client\{i\}} qI_i^j \tag{6}$$

A.2.3 Inputs quantity — qI (`Tot`)

Quantity of input needed for each output unit, given by a technical coefficient
Containing object: `Input`→ `Firm` → `Market` Elements used:

Element	Description	In Object	LSD
q (V)	quantity of output produced by the firm	`Firm`	`Quantity`
β_k (P)	technical coefficient of input k	`Input`	`quantity_coeff`

$$qI_k = \beta_k \cdot q \tag{7}$$

A.2.4 Final sales — Y^{S_F} (`FinalSales`)

Firm's sales to final demand. Computed as the market share for the final demand of the firm times total final demand for the market.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
ms_f (V)	firm's market share in the final market	`Firm`	`Ms_final`
D (V)	total demand in the final market	`Market`	`Demand`

$$Y^{S_F} = ms_f \cdot D \tag{8}$$

A.2.5 Final Demand — D (`Demand`)

Total final demand for the market (i.e. excluding business demand). It is computed as a smoothed adjustment between its own previous value and the current level of the target final demand. Markets not selling to final consumers have no final demand.
Containing object: `Market`
Elements used:

Element	Description	`In Object`	`LSD`
$isFinal$ (P)	Parameter indicating whether the sector serve final demand or not	`Market`	`IsFinal`
D^* (V)	Target final demand for the sector	`Market`	`TargetDemand`
D_{t-1} (V-1)	Lagged final demand	`Market`	`Demand`
s^D (P)	smoothing parameter for the demand	`Market`	`smoothDemand`

$$D = \begin{cases} s^D D_{t-1} + \left(1 - s^D\right) D^*_t & \text{, if } isFinal = 1 \\ 0 & \text{, if } isFinal = 0 \end{cases} \tag{9}$$

A.2.6 Target demand — D^* (`TargetDemand`)

Target level of final demand. It is determined as a function of the average quality level for each of the characteristics defining the product. The (inverse of) price is used as an added characteristic.
Containing object: `Market`
Elements used:

Element	Description	`In Object`	`LSD`
H (P)	constant	`Market`	`Constant`
$\overline{p}_{t-1}$ (V-1)	Average lagged price of the final sector	`Market`	`AvPrice`
$\overline{y}_m$ (V)	Average values of the quality m features	`Characteristic`	`AvValue`
α^p (P)	Parameter of sensitivity to price	`Market`	`alphaPrice`
α^y_m (P)	Parameter of sensitivity to quality features (one for each m feature of the good)	`Characteristic`	`alpha`

$$D^* = H \cdot \left(1/\overline{p}_{t-1}\right)^{\alpha^p} \prod_{m=1}^{M} \overline{y}_m^{\alpha^y_m} \tag{10}$$

A.2.7 Market share for final demand — ms_f (`Ms_final`)

Market share of a firm on the market for the final demand, if present. Computed as a smoothed adjustment of previous period value toward the target market share.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
ms_f^* (V)	Target market share for final demand	Firm	TargetMs
$ms_{f,t-1}$ (V-1)	Lagged market share for final demand	Firms	Ms_final
s^{MS} (P)	Smoothing parameter for the market share	Market	smoothMs

$$ms_f = s^{MS} ms_{t-1} + \left(1 - s^{MS}\right) ms_t^* \tag{11}$$

A.2.8 Target market share for final demand— ms^* (TargetMs)

Theoretical share of the final demand of the firm, to which actual shares slowly adjust. It is computed as the ratio between the quality index of the firm and the sum of all quality indexes of the firms in the market.
Containing object: Firm → Market Elements used:

Element	Description	In Object	LSD
I (V)	firms' competitiveness index	Firm	IndexQ
n_S (P)	Number of firms in the market	Market	(count)

$$ms^* = \frac{I}{\sum_{i=1}^{n_S} I_i} \tag{12}$$

A.2.9 Competitiveness index — I (IndexQ)

Quality index meant to represent the competitiveness of the firm in the final market. It is computed as the product of all quality levels of the firm's product, measured over the characteristics. The inverse of price acts as a further characteristic.
Containing object: Firm → Market
Elements used:

Element	Description	In Object	LSD
p_{t-1} (V)	price of firm output in the previous period	Firm	Price
y_m (V)	output features	OutCh	y
α^p (P)	parameter of sensitivity to price	Market	alphaPrice
α_m^y (P)	parameter of sensitivity to quality features (one for each m feature of the good)	Characteristic	alpha
M (P)	Number of characteristics defining the product in the sector	Market	- - -

$$I = (1/p_{t-1})^{\alpha^p} \prod_{m=1}^{M} y_m^{\alpha_m^y} \tag{13}$$

A.2.10 Average output price — $\overline{p}$ (`AvPrice`)

Average price for the final demand. The price is computed averaging single firms' prices weighted with their market shares of final demand.
Containing object: `Market`
Elements used:

Element	Description	In Object	LSD
p_i (V)	Price of the output of firm i	Firm	Price
$ms^i_{f,t-1}$ (V-1)	Previous period's market share of the final demand market for firm i.	Firm	Ms_final
n_S (P)	Number of firms in the market S	Market	- - -

$$\overline{p} = \sum_{i=1}^{n_S} p_i \cdot ms^i_{f,t-1} \tag{14}$$

A.2.11 Average quality — $\overline{q}_m$ (`AvValue`)

Average quality for each characteristic of products sold in a sector. Computed as the average of all products sold by firms wheighted with the market shares on the final market.
Containing object: `Characteristic` → `Market`
Elements used:

Element	Description	In Object	LSD
y^i_m (V)	quality of the feature m of the output produced by firm i	OutCh	y
$ms^i_{f,t-1}$ (V-1)	market share for final demand of firm i	Firm	Quantity
n_S (P)	Number of firms in the market S	Market	(count)

$$\overline{q}_m = \sum_{i=1}^{n_S} y^i_m \cdot ms^i_{f,t-1} \tag{15}$$

A.3 Production - Qualitative aspects

The quality of products sold in a market is defined over an exogenous number of product's characteristics. The level of quality of the product for a firm in respect of each characteristic is determined by two factors: quality of the inputs and competences of the firm. Competences represent the capacity of transforming each characteristic of each input in quality levels of the product.

The price of products is treated in the model as an added characteristic, but for the fact that it is determined in a different manner: price is computed as a mark-up over the costs of inputs.

The model can represent either "exogenous" inputs, obtained from sectors not represented in the model, and "endogenous" inputs, from sectors represented in the model. This second types of inputs are chosen by client firms every given number of periods, using a random choosing function with probabilities proportional to the average quality of the products in the input sector.

A.3.1 Quality — y_m (y)

Quality levels of the characteristic a firm's product. The number of characteristics must be identical for firms in each sector, while it can vary for firms in different sectors. Computed as the average of competencies times qualities of inputs.
Containing object: `OutCh` → `Output` → `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
$a_{m,i,h}$ (P)	Firm's competencies in producing quality level m using characteristic h of input i	`CompInCh`	`a`
$w_{i,h}$ (P)	Quality level of characteristic h for input i	`InCh`	`w`
$NInCh_i$ (P)	Number of characteristics of input i	`InCh`	(count)
NIn_S (P)	Number inputs used in the production of the output in sector S	`Input`	(count)

$$y_m = 1 + \frac{\sum_{i=1}^{NIn_S} \sum_{h=1}^{NInCh_i} a_{m,i,h} \cdot w_{i,h}}{\sum_{k=1}^{NIn_S} NInCh_k} \tag{16}$$

A.3.2 Supplier selection — Id^{Input} (IdInput)

Identification number of the supplier of the input. A firm consider whether to change supplier only every several time steps. When it does, it assigns to each potential supplier (i.e. firms in the supplying market) a probability of being chosen proportional to its index of competitiveness.

Note that, besides changing the Id^{Input}, the client firm updates also all the parameters of the quality levels of the input, and the price, of its input.
Contained in object `Input` → `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
I_S (P)	firms' competitiveness index.	`Firm` (in the input sector S)	`IndexQ`
Id_S^{Firm} (P)	identification number of firms.	`Firm` (in the input sector S)	`IdFirm`
t_{change} (V)	Value extracted from a uniform distribution on discrete values indicating time of reviewing suppliers. At $t = 1$ and every t when the firm changes its supplier t_{change} is extracted from the uniform $(t + t^{min}, t + t^{max})$	`Input`	`CounterDecision`

$$Id^{Input} = \begin{cases} Id_{t-1}^{Input} & \text{if } t \neq t_{change} \\ i,\ \Pr(i = Id^{Firm}) = \frac{I_i}{\sum_j I_j} & \text{if } t = t_{change} \end{cases} \tag{17}$$

A.4 Financial accounts

The model records the financial statuses of firms during their life times. The most important variable is the price of products, determined by firms using a mark-up on the variable costs. Firms record the following financial variables: revenues, profits, financial capital (i.e. cumulated net profits). Further extension of the model may exploit the financial data to implement, for example, investment capacities or a financial sector, currently not implemented.

A.4.1 Output price — p (`Price`)

Price of the final good as a mark-up decision of the single firm.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
mkp (P)	mark-up applied to total variable costs. (initially given, has to be endogenized in the firm decision as a function of input price, quality feature and market share change)	`Firm`	`mark-up`
c^V (V)	variable costs	`Firm`	`VariableCosts`
q (V)	quantity produced by each firm, used to obtain the unit variable cost	`Firm`	`Quantity`

$$p = \begin{cases} \dfrac{c^V}{q}(1 + mkp) & \text{if } q > 0 \\ \displaystyle\sum_{k=1}^{K} p_k^I \cdot \beta_k & \text{if } q = 0 \end{cases} \tag{18}$$

A.4.2 Variable costs — c^V (`VariableCosts`)

Variable costs, computed as the costs of inputs used for the production.
Contained in object `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
q_k^I(V)	quantity of inputs used	`Input`	`Tot`
p_k^I (V)	price of each input k, initially given (when the input is bought on a general input market; else, given by the output price of the supplier firm)	`Input`	`price_input`

$$c^V = \sum_{k=1}^{K} q_k^I \cdot p_k^I \tag{19}$$

A.4.3 Revenues — R (`Revenues`)

Total revenues from sales, computed as the price times sales.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
Y (V)	quantity sold by the firm (different from the quantity produced, depending on sectors)	`Firm`	`Sales`
p (V)	price of output	`Firm`	`Price`

$$R = Y \cdot p \tag{20}$$

A.4.4 Profits — π (`Profit`)

Profits (or losses), computed as the revenues minus variable costs and fixed costs, supposed exogenous.
Containing object: `Firm` → `Market`
Elements used:

Element	Description	In Object	LSD
R (V)	firm revenues	`Firm`	`Revenues`
c^V (V)	variable costs	`Firm`	`VariableCosts`
c^F (P)	fixed costs, initially given	`Firm`	`FixedCosts`

$$\pi = R - c^V - c^F \tag{21}$$