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BRIDGING THE GAP BETWEEN SCHUMPETERIAN COMPETITION AND EVOLUTIONARY GAME THEORY

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Abstract:
This paper suggests that the analysis of Schumpeterian competitionwithin the Nelson--Winter model should be complemented with evolutionary game theory. This model and its limitations for density-dependent Schumpeterian strategies are presented in terms of the equations of evolutionary dynamics. Formulated as evolutionary games, the set of strategies can easily be extended from innovators and imitators to routinists, complementors, and mixers. All strategies are presented in relation to a modified version of the Hawk-Dove Game. In this setting, the possibility of coexistence of several Schumpeterian strategies is proved. This is an example of Schumpeterian competition within evolutionary game theory.

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Bridging the gap between Schumpeterian competition and evolutionary game theory

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This paper suggests that the relatively concrete analysis of Schumpeterian competition in terms of the Nelson–Winter model should be complemented with evolutionary game theory. To prepare for this complement, the Nelson–Winter model is presented in terms of the equations of evolutionary dynamics that are important within evolutionary game theory. Thereby it is demonstrated that the model as serious limitations with respect to the study of the density-dependent success of Schumpeterian strategies. Furthermore, the set of analysed strategies has been limited. When Schumpeterian competition is defined in terms of evolutionary games, the set of strategies can easily be extended from including only innovators and imitators to the additional coverage of routinists, complementors, and mixers. All strategies are presented in relation to a modified version of John Maynard Smith's Hawk–Dove Game. In this setting, the possibility of coexistence of several of these Schumpeterian strategies is proved. This analysis is presented as an example of the way evolutionary game theory might complement the Nelson–Winter model.

Keywords: Schumpeterian competition; Schumpeterian strategies; Evolutionary game theory; Hawk–Dove game; Nelson–Winter models.

1. Introduction

This paper is written on the occasion of the 25th anniversary of two pioneering books: Richard Nelson and Sidney Winter's *An Evolutionary Theory of Economic Change* and John Maynard Smith's *Evolution and the Theory of Games*. Nelson and Winter presented a model that served to explore Schumpeterian competition while Maynard Smith outlined evolutionary game theory. However, the simultaneity of their publication dates is largely a matter of coincidence as it the fact that both contributions to the analysis of evolutionary dynamics were announced in papers from 1973 (Nelson and Winter; 1973; Maynard Smith and Price, 1973). Furthermore, two separate research traditions were established. The tradition of analysing Schumpeterian competition that relates to the Nelson–Winter model has reached a number of results with respect to both industrial dynamics and economic growth, but its main emphasis has been on routine behaviour and innovation within established firms. Therefore, it has received much attention in leading scientific journals within business economics, but the diversity of the literature means that it has not been surveyed. In contrast, the research tradition of evolutionary game theory within economics has paid little explicit attention to the problems emerging from the Schumpeterian vision. It has instead focussed on three other types of problem: the overcoming of the paradoxes emerging from the hyper-rationalism of classical game theory, the general analysis of density-dependent evolution of game strategies, and the explanation of basic human strategies of cooperation and conflict (Gintis, 2000; Bowles, 2004).

The present paper argues that, after 25 years with a large degree of separation, the two complementary research traditions should develop a systematic collaboration concerning the study of Schumpeterian competition. Seen from evolutionary game theory, a problem is that a rather limited set of problems have been covered by the cooperation between economists and biologists. In that context, the Schumpeterian vision may be seen as providing a challenging set of novel problems of evolutionary dynamics. Seen from the Nelson–Winter tradition, the addition of evolutionary game theory is even more advantageous for empirical studies, the construction of models, and the teaching of the results. The empirical advantage comes from the alliance between evolutionary game theory, economic psychology, and experimental economics. The modelling advantage is partly due to the fact that evolutionary game theory seems well suited for the analysis of the basic building blocks of Schumpeterian competition; for example, we may improve the study of the density-dependent success of strategies within the process of creative destruction. The pedagogical advantage is that a reformulation of the foundations of neo-Schumpeterian theory in terms of evolutionary game theory allows a systematic teaching of that theory, which has not been developed hitherto. Thus, there are good reasons for trying to overcome the gap between evolutionary game theory and Schumpeterian competition. This bridging has been prepared by Witt (2003) and others, but we are still missing a systematic cooperation. The paper sketches, in an admittedly naïve way, how the gap can be filled by starting with either evolutionary game theory or the Nelson–Winter model. The emphasis, however, will be on using evolutionary game theory to explore the strategies of Schumpeterian competition.

2. Game theory and evolutionary dynamics

The shortest way of characterising evolutionary game theory is to say that it is about density-dependent evolutionary processes. The density of a strategy is the share of agents that follow it in a population. When we know the density (or relative frequency) of all strategies, the payoff matrix of the game tells how each strategy will succeed. We can then study the evolutionary dynamics of the game. This approach is of crucial importance for studying the dynamics of Schumpeterian competition.

Evolutionary game theory differs radically from the classical game theory developed by John von Neumann and Oskar Morgenstern (1944). This version of game theory did not help Schumpeter to express his evolutionary vision because it deals with the strategic interactions of hyper-rational agents. Since such agents immediately find their best strategies, their interactions does not take the form of an evolutionary process. Therefore, we can move directly to the analysis of the equilibrium outcome of the games of the rational agents: the evolution is at best mimicked in their brains.

Although John Nash extended the analysis to the equilibria of non-cooperative games, he opened up for the inclusion of boundedly rational agents and thus for the treatment of evolutionary processes. Unfortunately, this possibility was only described in Nash's (1996, 32–33) PhD thesis, while the editors excluded it from the published version. Furthermore, economists were so accustomed to the assumption of hyper-rational agents that they ignored the serious treatment of other possibilities. Therefore, the evolutionary biologist John Maynard Smith had to reinvent the possibility. The result was an evolutionary version of the Nash Equilibrium, the Evolutionarily Stable Strategy. To formulate this equilibrium concept, Maynard Smith (1982) assumed a agents with fixed strategies. Here the relevant unit of analysis of a population described by relative frequencies of the different strategies. Here the frequency of a particular strategy changes according to its payoff relative to other strategies: if it has a supernormal payoff, its frequency increases; if it has a subnormal payoff, its frequency decreases. To study the evolutionary stability of a population in which all members follow a particular strategy, we simply ask whether a few "mutants" with a deviating strategy can obtain supernormal payoffs. If this is *not* the case, the incumbent strategy is an Evolutionarily Stable Strategy. Such a situation does not necessarily imply that all agents are applying the same strategy. An Evolutionarily Stable Strategy may also be "mixed" in the sense that stability within the population implies the coexistence of two or more strategies.

Maynard Smith (1982, vii) prefaced his analysis by remarking: "Paradoxically, it has turned out that game theory is more readily applied to biology than to the field of economic behaviour for which it was originally designed." The background for this remark is that biologists deal with genetically coded agents with no rationality. Instead of the selection of strategies by hyper-rational agents, they study natural selection based on the frequency-dependent payoffs of the strategies. Therefore, they do not encounter hyper-rational agents whose deliberations move into infinite regress and who are unsure about the multiple equilibria of the games they are playing. The same simplification can be obtained by assuming agents with bounded rationality and the related routinised behaviour that changes infrequently. Since the members of the species *Homo sapiens* are such agents, evolutionary game theory has proved appropriate not only for biologists but also for economists (Gintis, 2000; Bowles, 2004). This explains why evolutionary game theory has spread rather widely among mathematically oriented economists (Weibull, 1995; Samuelson, 1997; Hofbauer and Sigmund, 1998).

Like classical game theory, evolutionary game theory starts from the specification of the set of agents, the set of strategies for each agent, and the payoffs to each agent for every possible list of strategy choices by the other agents. There are three major differences. First, the game takes place in a population of agents. Second, the game is, in principle, repeated forever with ever-changing combinations of agents. Third, the payoff of a particular strategy at time t influences its frequency at time $t+1$. This evolutionary mechanism can be defined and interpreted in different ways. The simplest version is the so-called replicator dynamics for large populations, which was already hinted at above. Here the change of the frequency of a strategy is determined by its performance relative to the average performance:

$$\text{Change of frequency of } i = \text{Frequency of } i \times (\text{Payoff of } i - \text{Average payoff}).$$

Let us be more formal. Each agent plays against a randomly chosen opponent at each step of the game. The payoff of an agent following strategy i against an agent with strategy j is found in as the a_{ij} entry of the payoff matrix. The expected payoff is determined by the frequencies of strategies, where x_i is the frequency of strategy i (and $\sum_{i=1}^{n} = 1$). Thus the expected payoff of strategy i is $\pi_i = \sum_{j=1}^{n} x_j a_{ij}$. Similarly, we can calculate the average payoff as $\pi = \sum_{i=1}^{n} x_i \pi_i$. These definitions allow us to specify the

Game Dynamics Equation, which is also called the *Replicator Equation*:

$$\frac{dx_i}{dt} = x_i(\pi_i - \pi), \text{ for } i = 1, \ldots, n. \tag{1}$$

This equation can be interpreted in different ways. In biology, π_i is interpreted at the fitness of strategy i, i.e. as the number of surviving descendants in the next generation. For the present purposes, the simplest interpretation that agents imitate the strategies of more successful opponents. In the continuous case, this switching will secure that the average performance of the population reaches a stationary state. However, the agents might not consider their strategy in each period. Therefore, we could add the speed of adaptation α so that the time derivative of the frequency of strategy i is $\alpha x_i(\pi_i - \pi)$.

The replicator equation is sufficient for exploring the types of evolutionary dynamics that are of interest to Maynard Smith and many other contributors to evolutionary game theory. This equation secures that the dynamics moves the population to a Nash Equilibrium of the game, but this is not necessarily an Evolutionarily Stable Equilibrium (Gintis, Gintis (2000), Ch. 9; Hofbauer and Sigmund, 1998, Ch. 7). This is one of the reasons why many studies add an explicit treatment of the complementary dynamics that are created by "mutation", or innovation. The formalisation of this aspect of evolutionary game dynamics is of especial importance for the treatment of the Schumpeterian vision. More generally, the task is to confront the problem of how strategies that are not present in the population can emerge. In evolutionary biology, the major mechanism is mutation of the DNA that codes the strategy. The corresponding mechanism in evolutionary economics is the infrequent and random change of strategy. But here there are many other possibilities. However, the general problem is to define a "mutation" matrix in which q_{ji} is the probability in each period that an agent with strategy j spontaneously switches to strategy i. These "mutation" probabilities are not necessarily constant. Their contribution to the evolutionary dynamics is defined by the *Mutator Equation*:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} x_i q_{ji} - \pi x_i, \text{ for } i = 1, \ldots, n, \tag{2}$$

where the term $-\pi x_i$ ensures that the sum of the frequencies stays equal to 1.

Both equations (2) and (1) can be derived from the *Replicator–Mutator Equation*:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} x_j \pi_j q_{ji} - \pi x_i, \text{ for } i = 1, \ldots, n. \tag{3}$$

This equation (3) provides the general description of the evolutionary dynamics that can take place both by means of frequency-dependent selection and by mutational change (Page and Nowak, 2002). If we eliminate frequency-dependant selection (by setting $\pi_j = 1$ for all j), we obtain equation (1). Similarly, we obtain equation (2) by eliminating mutation ($q_{ji} = 1$ for $j = i$ and $q_{ji} = 0$ for $j \neq i$). These relationships are depicted by Figure 1. This figure emphasises the central place of the Replicator–Mutator Equation. The figure adds that we sometimes want to study evolutionary dynamics in the short run. This is done by means of Price's Equation, which is presented below as equation (4). Price's Equation emphasises the statistical nature of evolutionary change.

Especially in its extended form, evolutionary game theory provides a general description of evolutionary dynamics that includes both biological and cultural change

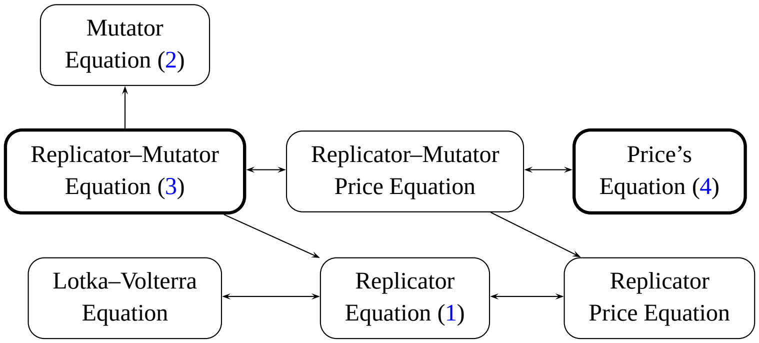


Figure 1: Equations of evolutionary dynamics (based on Page and Nowak, 2002, 94)

(Nowak, 2006). Actually, one might argue that those who do not study games in this meaning are covering special rather than general cases. For instance, classic game theory relies heavily on the very special case of fully rational and fully informed agents. This case not only leads to a multitude of equilibrium solutions from which agents can hardly select any preferred ones but also to embarrassments when it comes to laboratory tests by means of experimental economics (Kreps, 1990; Mailath, 1998). It has, therefore, been obvious to confront these problems when evolutionary game theory was transferred from evolutionary biology to economics. But the method of evolutionary game theory is different from and more complex that that of classic game theory. This method copes not only with the specification of elementary contests (games) but also with the way boundedly rational agents learn from and invent new behavioural patterns within the overall and long-term series of contests in the population. Here the exploitation of new business opportunities is characterised by a "competence–difficulty gap" (Heiner, 1989) that necessitates rule-of-thumb behaviour.

3. The Nelson–Winter model of Schumpeterian competition

The density-dependent evolutionary process is also a prominent feature of the Nelson and Winter's (1982, Chs 12–14) model of Schumpeterian competition. However, the basic similarities between this model and models framed within evolutionary game theory has normally been overlooked. The reason is largely that different modelling styles. The starting point of Nelson and Winter (1982, 29) was that "Schumpeter's basic contributions have been widely invoked by economists in their verbal accounts of the behavior of industries, but have received only few attempts at formalization." Their problem was that "the intellectual coherence and power of thinking about Schumpeterian competition have been quite low, as one would expect in the absence of a well-articulated theoretical structure to guide and connect research." On this background, they stated (p. 39) that "we are evolutionary theorists *for the sake* of being neo-Schumpeterians—that is, because evolutionary ideas provide a workable approach to the problem of elaborating and formalizing the Schumpeterian view of capitalism as an engine of progressive change." This purpose led to the description of economic evolution as a process in which firms follow rules or routines that can occasionally be mutated or adapted. The standard case is an industry (or an economy) where new process techniques are introduced and imitated. This case had defined a paradigm of further research on the conditions of R&D as a determinant of industrial concentration, dynamic competition in alternative technological regimes, the relationship between innovators and imitators, and even on evolutionary growth theory. However, the specialised nature of the modelling tradition implies serious constraints and, in addition, makes it hard to teach. This situation might be changed by relating the Nelson–Winter tradition to evolutionary game theory and to the basic equations of evolutionary dynamics.

Like evolutionary game theory, the Nelson–Winter model depicts an elementary contest (a game) in which boundedly rational agents learn from and invent new behavioural patterns within the overall and long-term series of contests in the population. Here the exploitation of new business opportunities is characterised by a "competence–difficulty gap" (Heiner, 1989) that necessitates rule-of-thumb behaviour. This behaviour leads to a process of selection between firms competing with different technologies in an industry producing a homogeneous product. These firms receive different profits that are invested in productive capacity. Firms with a supernormal profit rate expand while those with subnormal profits contract. Thereby, the selection dynamics is described by the Replicator Equation (1) for their capacity shares. Nelson and Winter (1982, 243) also explored the statistical nature of replicator dynamics. They focussed on the average productivity p of the industry and used the replicator equation to study its time derivative (dp/dt). Like R. A. Fisher (1999) had done previously, they demonstrated that this derivative is proportional to the variance of the capacity-share weighted individual productivities. Since the weighted productivity is $x_i p_i$, the core variable is $\mathrm{Var}(x_i p_i)$.

To proceed to the general case with a less than perfect selection as well as with change of the productivities of the firms, we need a basic decomposition of evolutionary change that was invented by the evolutionary biologist George R. Price (Frank, 1998; Andersen, 2004). Applied to the Nelson–Winter model, the average productivity change is determined by the sum of the selection effect and the 'mutation' effect. The selection effect is the covariance between the payoffs ($\mathrm{Cov}(\pi_i, p_i)$) and the productivities and the mutation effect is the average of the weighted firm-internal change of productivity ($\mathrm{E}(\frac{dp_i}{dt})$). These effects are combined in *Price's Equation*:

$$\frac{dp}{dt} = \sum_i x_i \pi_i p_i - \pi p + \sum_i x_i \frac{dp_i}{dt} = \mathrm{Cov}(\pi_i, p_i) + \mathrm{E}(\frac{dp_i}{dt}). \tag{4}$$

Price's Equation (4) can even be used to reveal two characteristics of the fully developed Nelson–Winter model. The first characteristic is that Nelson and Winter chose to deviate from the pure Replicator Equation. The reason is that the application of this equation to a population of firms with different but fixed productivities leads to monopoly. Therefore, they modelled the industrial dynamics according to Cournot's model of oligopoly. Augustin Cournot had, according to Schumpeter (1954a, 76n), "one of the best minds that ever occupied themselves with our discipline". He even discovered the Nash Equilibrium of his oligopoly game by studying the dynamics of the game (Cournot, 1971, Ch. 7). However, Schumpeter abstained from following this lead because the overwhelming majority of economists did not believe in Cournot's *tâtonnement* process. The opinion was milder when Nelson and Winter developed their model, so they included a Cournot process for the production capacities of their firms. Each firm makes the conjecture that the production capacities of all other firms will stay constant. On this background they consider themselves monopolists with respect to the remainder of the demand curve—and modify their capacity for the next period accordingly. The resulting dynamics for an appropriately chosen demand curve is that small firms largely behave according to the Replicator Equation while large firms show investment restraint (Nelson and Winter, 1982, equation 7 on page 302). Therefore, several firms survive when the Nash Equilibrium is reached—even if they have different productivities.

The second characteristic of the Nelson–Winter model is that the mutation effect is more important than the selection effect. The explanation is that they use their version of the Cournot process as a test bed for the different strategies of Schumpeterian competition. Here the immediate focus is on the dynamics of the firm rather than on the dynamics of the frequencies of strategies or technologies (see, however, the alternative evolutionary perspective by Winter, 1987). In the simplest case, the dynamics start in a Nash Equilibrium for the sustainable number of firms with given technologies. Then the firms are allowed to perform innovative and/or imitative activities with probabilistic results. In general, firms that obtain better productivities earns a profit and adapt their capacity. However, since firms act as Cournot oligopolists, the potential of expansion is not necessarily exploited. Actually, a firm with a very high market will want to contract its capacity in order to maximise its short-term profit after obtaining an improved productivity. Although the result of this mechanism is a higher average productivity of the industry, it is rather obtained by the mutation effect than the selection effect. In this respect, the Nelson–Winter model differs from Schumpeter Mark I in which an innovative period during which an innovation is introduced by new firms is followed by a period in which the selection effect becomes predominant. Instead, they adhere to Schumpeter Mark II that is characterised by repeated innovations in a relatively stable population of large firms. Their major contribution is to formulate this model in terms of firms that act according to the Cournot conjecture of the short-term stability of the output of other firms. However, Winter (1984) and several followers have demonstrated that this type of model can also to some extent cover the early phases of the dynamics of an industry according to Schumpeter Mark I with industry creation due to innovative entry.

Since the focus of the Nelson–Winter model is on the firm, we cannot directly use the Mutator Equation (2), but in a discrete technology space we may reinterpret the mutation matrix so that q_{ijk} is the probability in each period that firm i with technology j switches to technology k. This probability is not constant. On the contrary, it is determined by the firm's current level of capacity, its R& D strategy, and the general technological regime. A firm's R& D strategy is to use a fraction of its current expenses on innovative research and/or imitative research. The technological regime determines the probabilistic productivity of these research expenses. The outcome of the R& D of a period is an innovative result and/or an imitative result. These results are compared with the current productivity, and the firm chooses the best alternative. This mutation behaviour allows Nelson and Winter and their followers to make a relatively rich study of the functioning of the Schumpeterian strategies of innovation and imitation as well as mixes of these strategies. They have also been able to define important combinations of the parameters that determine several of basic technological regimes that govern industrial dynamics. A natural extension of these efforts is to try to overcome the gap between the fairly abstract issues of the Nelson–Winter model and the concrete issues confronted by empirical researchers. Thus, Malerba et al. (1999, 3–6) argue that while the first generation of models of Schumpeterian competition has largely been characterised by an attempt to understand the basic logic of evolutionary processes, the second generation should be "history-friendly" models.

However, we may also imagine a third generation of models that return to the basic issues of Schumpeterian competition and relate them to evolutionary game theory. Several of these issues have been ignored because the literature on Schumpeterian competition has been constrained by the underlying Cournot-inspired model. The basic function of the investment restraint of this model is that firms do not follow their advantageous strategy to its radical conclusion, i.e. to its monopolistic status. For example, although the imitating strategy has lower R&D costs than the innovating strategy, innovators are not driven to extinction within the population of firms. This approach excludes the focussed analysis of the density-dependent success of the different strategies. Furthermore, the concentration on the analysis of innovation that related to the supply of an industry has removed attention from strategies that relate to the change of the preferences of consumers (Andersen, 2007). For these, and other reasons, the set of analysed strategies has been limited. It is a major task to extend the set of analysable Schumpeterian strategies.

4. The structure and strategies of Schumpeterian games

In order to develop evolutionary games of Schumpeterian competition we shall move from the analysis of the detailed evolutionary dynamics of a Cournot game in a single industry to an evolutionary process that includes Cournot games in a huge number of industries. By means of these games, we shall depict the long-term evolution of the economic system in terms of the emergence and disappearance of a large number of industries or, better, economic niches. Each niche emerges as a business opportunity that is exploited by a number of firms, and their payoffs can be accounted for after the niche has disappeared. These payoffs depend to a large degree on Schumpeterian strategies that were fixed at the time when the firms entered the new niche market. The dynamics of such a game may lead to an evolutionarily stable mix of Schumpeterian strategies.

This suggested process can, surprisingly, be analysed in relation to Maynard Smith's (1982) pioneering book. This book presents the analytical apparatus of evolutionary game theory and exemplifies its use by explaining animal behaviour. Both the analysis and the examples focus on the Hawk–Dove Game in a large number of variants. In its basic form, it concerns bilateral contests over resources. The Hawk strategy is to try to win the resource, if necessary at considerable costs. The Dove strategy is to share the resource when meeting another Dove and to abandon if the opponent is a Hawk. The payoff matrix of the game is specified so that an Evolutionarily Stable Strategy is obtained by a mixed population of Hawks and Doves. Thereby, Maynard Smith was able to explain a behavioural dichotomy that is found in many natural populations. By apparently small modifications of the Hawk–Dove game, he was also able to explain many other facts about animal behaviour. However, the payoff matrix of the basic version of this game is depicted in Figure 2, which only depicts the payoffs of the focal player. This player's strategy is depicted by the rows while his opponent's strategy is depicted by the columns. In the figure $\pi(H,H)$ is the focal firm's payoff when both players are Hawks; $\pi(H,D)$ is the payoff of playing Hawk against a Dove, $\pi(D,H)$ is the payoff of playing Dove against a Hawk, and $\pi(D,D)$ is the payoff of playing

		Opponent firm	
		Hawk	Dove
Focal firm	Hawk	$\pi(H,H)$	$\pi(H,D)$
	Dove	$\pi(D,H)$	$\pi(D,D)$

Figure 2: The basic setup of the Hawk–Dove Game

Dove against a Dove. The standard Hawk–Dove Game presupposes that $\pi(H,D) > \pi(D,D) > \pi(D,H) > \pi(H,H)$.

How can the Hawk–Dove Game be applied to explore a Schumpeterian game of exploiting new business opportunities? The starting point is a large population of economic players (firms) that are engaged in frequent contests over new business opportunities. According to Schumpeter, such contests involve groups of players that “swarm” around different business opportunities. However, we shall assume that only two players become involved in the exploitation of each niche opportunity. These duopolists are drawn at random from the total population of firms. Before entering the contest for a niche, each of the two players has committed himself to a particular Schumpeterian strategy. The reason is that such a strategy is something deeply rooted in the structure and routines of the firm. The consequence is that there is no chance of adapting the strategy after entering a particular contest for a niche. Furthermore, the participation in the contest involves an investment that has to be made before the strategy of the opponent is known.

Each Schumpeterian contest for a niche takes place in three stages:

- In the first stage each player uses his given strategy to determine how much and which types of resources to invest for the exploitation of the new business opportunity. The size of the investment differs between the strategies, e.g. the fixed costs of an Innovator, $F(I)$, differs from those of a Routinist, $F(R)$. When the investment has been made, it represent sunk costs for the player (cf. Sutton, 1998).

- The second stage of a contest may be considered as a duopoly play that is obviously more complex than the games studied in the Cournot tradition. One difference is that we shall assume that the second stage of the game continues until the business opportunity has become unprofitable and thus has disappeared from the economic horizon. Another difference is that the value of the second stage depend on the innovative investments of both contestants: larger investments lead to larger values (apart from the case of duplicated investments). Yet another difference is that there is not always an equal sharing of the value between the two contestants. In the case of e.g. a contest between an Innovator and a Routinist, the Innovator gets all the value and the Routinist gets nothing.

- In the third stage each player studies his outcome of the duopoly play, i.e. his payoff minus the investment made in the first stage of the contest. Based on a comparison with the performance of other players (in parallel contests), the player considers whether a strategy change is called for. But because of the stickiness of strategies, the actual change of strategy will only be made infrequently. This is modelled in each round by the random selection of players that reconsider their strategy.

Strategy	Description
Innovator	The player wants to secure the niche for himself
Routinist	The player only exploits the niche to a very limited extent and can coexist with another Routinist
Exploiter	The player tries to imitate the other player at low costs
Complementor	The player tries to expand the niche by complementing the efforts of the other player
Mixer	In each contest, the player selects a strategy by stochastic choice

Table 1: A set of Schumpeterian strategies

Let us postpone the long term dynamics to the next section and only consider the set of individual games that take place within a single period. The major task is to specify a relevant set of strategies and their payoffs. More specifically, we shall try to define five Schumpeterian strategies (summarised in Table 1). The Innovator is a Hawk who wants to secure the exploitation of the niche for himself by making a heavy investment before entry. In contrast, the Routinist only makes a small pre-entry investment that allows a Dove-like coexistence with another Routinist. Three other strategies cannot be interpreted as Hawk or Dove. The Exploiter makes a relatively small investment that allows him to imitate the efforts of an opponent Innovator. The Complementor's investment serves as a preparation for expanding the size of the niche. Finally, the Mixer uses a combination of other strategies in the exploitation of the niche.

The specification of the strategies of Table 1 can most easily be made in relation to a sequence of gradually more complex Schumpeterian games. The most basic of these games is the, already mentioned, Innovator–Routinist Game, which relates loosely to be the one implied by Schumpeter (1934, Ch. 2) basic model. According to the Innovator strategy a new business opportunity should be extensively exploited by means of a heavy outlay of fixed costs F, which after they have been made represent sunk costs for the Innovator. The investment leads to a business activity with the expected net value of V, but whether the Innovator can appropriate this value depends on the strategy of the opponent in the contest. According to the Routinist strategy the new business opportunity should be exploited in a cautious manner. This implies that the initial investment is only a fraction of the Innovators investment (F/b_R, where $b_R > 1$) of the investment of the pioneer. The consequence is that the contribution of an adaptionist to the expected net value of the second stage of the contest is also smaller that that of the pioneer (V/a_R, where $a_R > 1$).

We now turn to the size of the payoffs of the four strategy combinations of the Innovator–Routinist Game, as depicted in Figure 3(a). When an Innovator meets a Routinist, it is the Innovator who gets the whole net value V. We thus have $\pi(I,R) = V - F$. Similarly, when a Routinist meets an Innovator, the Routinist gets no share in the value so $\pi(R,I) = -F/b_R$. However, when an Innovator meets an Innovator, there is a duplication of effort so that the expected net value of the second stage of the game is only equal to the result of the investments of a single firm. The reason is that they duplicate the innovative effort—so the value of the second stage of a contest does not increase compared to a contest between an Innovator and a Routinist. This means that each firm can expect to obtain the value of $V/2$ and its payoff is $\pi(I,I) = V/2 - F$.

	I	R
I	$V/2-F$	$V-F$
R	$-F/b_R$	$V/a_R-F/b_R$

(a)

	I	R
I	-8	4
R	-2	1

(b)

Figure 3: The Innovator–Routinist Game: (a) Structure of payoffs. (b) Numerical example of payoffs.

Similarly, the efforts of two Routinists represent another example of duplication so that they share the small value created by a single Routinist investment. Thus the focal player gets $\pi(R,R) = V/(2a_R) - F/b_R$.

Since it is the relationships between the payoffs that matter for the evolutionary process, we can just as well analyse the game by means of a numerical example. We may, for instance, let $V = 24$, $F = 20$, $a_R = 4$, and $b_R = 10$. The result is shown in Figure 3(b). What matters for the qualitative evolution of the game is that $\pi(I,R) > \pi(R,R) > \pi(R,I) > \pi(I,I)$—like in the Hawk–Dove Game. The assumption means that Innovators perform well against Routinists but bad against each other. Routinists have in-between payoffs, but Routinists perform better against each other than against Innovators. This seems to correspond to the stories given by Schumpeter about the exhaustion of entrepreneurial initiative.

We now turn to the population of players engaged in the Innovator–Routinist Game. At the beginning of step t of this game all players have chosen their strategy. Therefore, we know the frequencies of the strategies, I_t and R_t. These frequencies define a population-level strategy profile, which we shall call $mix_t = (I_t, R_t)$. The frequencies have to sum to one so that $I_t + R_t = 1$. This means that the population state can be characterised by I_t alone. Since the population is large we shall, furthermore, ignore the effect of the choice of the focal player on I_t. Therefore, the chance of the focal firm of playing against an Innovator is I_t and the chance of playing against a Routinist is $1 - I_t$. In the numerical example of Figure 3(b) the expected payoff of playing an Innovator is $\pi(I, mix_t) = -8I_t + 4(1 - I_t)$. Similarly, the payoff of playing Routinist is $\pi(R, mix_t) = -2I_t + 1 - I_t$.

When Schumpeter (1934, Ch. 6) turns to a more systematic account for economic and capitalist development, it becomes obvious that there are other ways of exploiting a Schumpeterian game than being an Innovator or a Routinist. The most obvious alternative is to be an Exploiter (or imitator), which is an important part of Schumpeter's "swarming" process around new business opportunities. The Exploiter strategy is not costless because the firm needs many capabilities for imitating the results of an Innovator. Thus the Exploiter strategy that is specialised for obtaining results from Innovators and will only survive if there is a significant number of Innovators to imitate. But there is also a less parasitic way of interacting with Innovators (and Routinists) in the "swarming" process. The Complementor strategy tries to complement the efforts of the other player by expanding the business niche. The effect on the opponent depends on how the niche relates to the core of the business opportunity, but we shall assume a symbiosis between Complementors and Innovators. Such a possibility is e.g. dealt with in several of the historical parts of Schumpeter's (1939) *Business Cycles* (see also Andersen, 2002).

Let us consider the addition of Exploiters to the Innovator–Routinist Game. The

	I	R	E
I	-8	4	-8
R	-2	1	1
E	7	-2	-2

(a)

	I	R	C
I	-8	4	4
R	-2	1	-2
C	16/3	-2/3	-11/3

(b)

Figure 4: Two extensions of the Innovator–Routinist Game: (a) The Innovator–Routinist–Exploiter Game. (b) The Innovator–Routinist–Complementor Game.

Exploiter strategy is denoted E while the frequency of Exploiters at time t is denoted E_t. This strategy is designed for exploiting contests with Innovators. For exploiting these possibilities the imitator needs to make an investment that is smaller than that of the Innovator but larger than that of the Routinist. This we obtain be specifying the parameter b_E, where $b_R > b_E > 1$ and by defining the initial investment as F/b_E. The Exploiter firm does not contribute anything to the net value of the second stage of the contest, but in contests it gets half of the expected net value. Take, for instance, a contest between an Exploiter and an Innovator. Here the Exploiter gets $\pi(E,I) = V/2 - F/b_E$. In Figure 4(a) we see a numerical example of an Innovator–Routinist–Exploiter Game, where $b_E = 4$ and the rest of the payoffs is taken from Figure 3. The calculation of the population-level payoffs of the strategies of this game has to take into account that $mix_t = (I_t, R_t, E_t)$.

Our final pure strategy is designed to complement the investments made by the other player in a pairwise competition—especially Innovators. We shall call this the Complementor strategy, using a term applied by Nalebuff and Brandenburger (1996) in a somewhat different context. We denote this strategy by C and its frequency by C_t. Its initial investment is determined by the parameter b_C so that $b_I > b_C > 1$ and initial investment is F/b_C. Thus we have that $F(I) > F(C) > F(E) > F(R)$. The contribution of the Complementor to the net value of a contest is interesting not only because it is of significant size (but not as large as that of the Innovator) but also because it in contests with Innovators does not duplicate but add a value that it itself appropriates, while it leaves the opponent's contribution to himself. The size of the complement is a fraction of that of the Innovator, so that $\pi(C,I) = V/a_C - F/b_C$ and $\pi(I,C) = V - F$. It is not so obvious what the Complementor strategy does in contests with Routinists. The reason is that there is little to complement. Instead we shall assume that the Complementor performs slightly better than the Routinist and takes the whole net value. Thus we have $\pi(C,R) = V/a_C - F/b_C$ and $\pi(R,C) = -F/b_R$. In contests between Complementors we shall assume that the situation is like in contests with Routinists, but the complementors have to share the value. Figure 4(b) gives an example of a Innovator–Routinist–Complementor Game, where $a_C = 2$ and $b_C = 3$. Here the state of the population is defined by $mix_t = (I_t, R_t, C_t)$.

Until now we have—in accordance with Schumpeter Mark I—assumed that players can only play pure strategies in the Schumpeterian game. There are good organisation theoretic reasons for this but, especially in the large firms of Schumpeter Mark II, we are likely to encounter mixed behavioural strategies. For instance, an obvious way of tackling the Innovator–Routinist Game is to play the Mixer strategy, i.e. to play different pure strategies with different probabilities. For instance, we may have a firm that determines its behaviour randomly in a way that makes it an Innovator in half of the

	I	R	$M^{1/2}$
I	-8	4	-2
R	-2	1	-1/2
$M^{1/2}$	-5	5/2	-5/4

Figure 5: An Innovator–Routinist–Mixer Game

contests and a Routinist in the other half of the contests (see Figure 5). More generally, it plays strategy I with probability p and strategy R with probability $1-p$. This possibility might have been recognised by Schumpeter (1942/42, Ch. 8), but he never made a systematic treatment of the possibility. The Mixer strategy presupposes that the firm has capabilities for different types of investment and that it can handle the internal inconsistencies that may arise from the contrasting types of behaviour. However, to stick to our assumption about the stickiness of strategies, we shall only allow a fixed composition of the mix, or a slowly changing mix that can mimic the strategy composition of the population. In the fixed case we denote the strategy M^p. It's expected payoff is the p-weighted average of the included pure strategies. In the population-level Innovator–Routinist–Mixer Game, the state is defined by $mix_t = (I_t, R_t, M_t^p)$.

The presentations of Exploiters, Complementors, and Mixers have been made as three simple extensions of the Innovator–Routinist Game. However, more complex games can also be defined. Space does not allow the presentation and evaluation of these further extensions. It should be noted that Mixers are not necessarily relevant in all situations. In contrast, the Innovator–Routinist–Exploiter–Complementor Game is perfectly feasible. However, when we turn to the evolutionary dynamics of this and other games, it turns out that not all strategies will survive.

5. The evolutionary dynamics of Schumpeterian games

The design of the above series of Schumpeterian games is, like Maynard Smith's Hawk–Dove Game, made to prepare for the evolutionary analysis of them. However, it is only when we study the evolutionary dynamics that the difference between classical game theory and evolutionary game theory becomes clear. Let us start by considering the method of evolutionary game theory at some length in relation to the analysis of the dynamics of Innovator–Routinist Game. This method begins with the contests between Innovators (I) and Routinists (R) at a particular step of the game, t. The task is to proceed to the dynamics of the frequencies of Innovators (I_t) and Routinists (R_t). Thus the state variable is the population-level strategy profile, $mix_t = (I_t, R_t)$. Here the frequencies sum to one. Thus an increase of a strategy's population share measured at time $t+1$ must be mirrored by a compensating decrease of the share of the other strategy.

For each of the many individual three-stage contests at time t, two players are chosen at random. Therefore, a focal player can expect an opponent to be an Innovator with probability I_t and a Routinist with probability R_t. Thus the average performance of a strategy depends on the composition of the population at that time. The expected payoffs for the focal player of playing the two strategies are

$$\begin{aligned} \pi(I, mix_t) &= \pi(I,I)I_t + \pi(I,R)R_t \\ \pi(R, mix_t) &= \pi(R,I)I_t + \pi(R,R)R_t \end{aligned} \tag{5}$$

In equations (5) we e.g. see that the expected outcome of playing Innovator is equal to the payoff of playing against an Innovator times the probability of meeting an Innovator plus the payoff of playing against a Routinist times the probability of meeting a Routinist. To evaluate the performance of the two strategies, we have to calculate the mean payoff:

$$\pi(mix_t) = \pi(I, mix_t)I_t + \pi(R, mix_t)(1 - I_t). \tag{6}$$

The change of the frequencies of the two strategies is determined by comparisons between their expected payoffs ($\pi(I, mix_t)$, $\pi(R, mix_t)$) and the average payoff, $\pi(mix_t)$. Figure 3 demonstrates that if the population is dominated by Routinists, Innovators will show above-average performance. In contrast, Innovators perform below average in a population dominated by Innovators. This means that those players who reconsider their strategies after a step of the game react differently in the two situations. In any case, a disequilibrated strategy profile of the population implies that $I_t \neq I_{t+1}$ and, correspondingly, that $R_t \neq R_{t+1}$. If the Innovator strategy I has become more frequent, then the probability that a focal player will meet an I player will increase. This his will lead to changed average performance of the strategies at time $t+2$ and, possibly, to further changes of the frequencies of the strategies.

The suggested type of evolutionary dynamics can most easily be defined in terms of the Replicator Equation (1). Here the speed of change is proportional to the distance between the expected payoff of the strategy and the mean payoff of the whole population. Thus we can use equations (5) and (6) to obtain

$$\begin{aligned} \frac{dI_t}{dt} &= \alpha I_t[\pi(I, mix_t) - \pi(mix_t)] \\ \frac{dR_t}{dt} &= \alpha R_t[\pi(R, mix_t) - \pi(mix_t)] \end{aligned} \tag{7}$$

where e.g. dI_t/dt is the speed of change of the share of Innovators and α is the responsiveness of strategy change to payoff differences.

According to equations (7), we have a stable population profile when $dI_t/dt = dR_t/dt = 0$. But in evolutionary analyses we are only interested in population profiles that are stable against small perturbations or mutations, i.e. evolutionarily stable population profiles. Thus we are only interested in Nash equilibria like $I_t = 0$ and $R_t = 0$ if they are evolutionarily stable. This stability is easy to check even without the explicit inclusion of the Mutator Equation (2). A population with only Routinists is not evolutionarily stable since the emergence of a few Innovators would demonstrate the superiority of this strategy (in a population dominated by Routinists). The evolutionarily stable population strategy is an appropriate mix of Routinists and Innovators. This result is depicted by Figure 6. On the horizontal axis we measure the frequency of I from left to right and the frequency of R from right to left. Thus when $I_t = 0$, then $R_t = 1$. The solid lines show the payoffs for Innovators and Routinists, while the dashed line shows the mean payoff. When there, for example, are few Innovators and many Routinists, the Innovator strategy performs better than the Routinist strategy. Therefore, the Innovator strategy will increase in frequency and I_t will move to the right. When the frequency have reached I_t^*, the payoffs of the two strategies are exactly the same and there will be no change. Here we have a Nash Equilibrium. In our numerical example of the Innovator–Routinist Game (Figure 3(b)), this equilibrium is obtained when $I_t^* = 1/3$

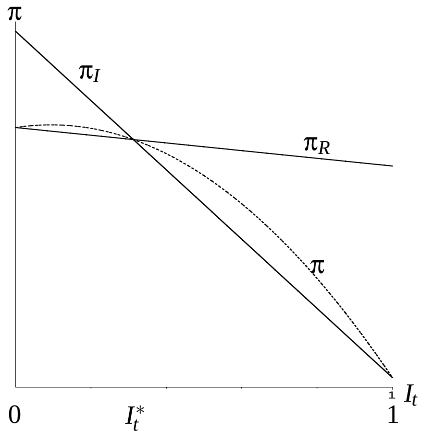


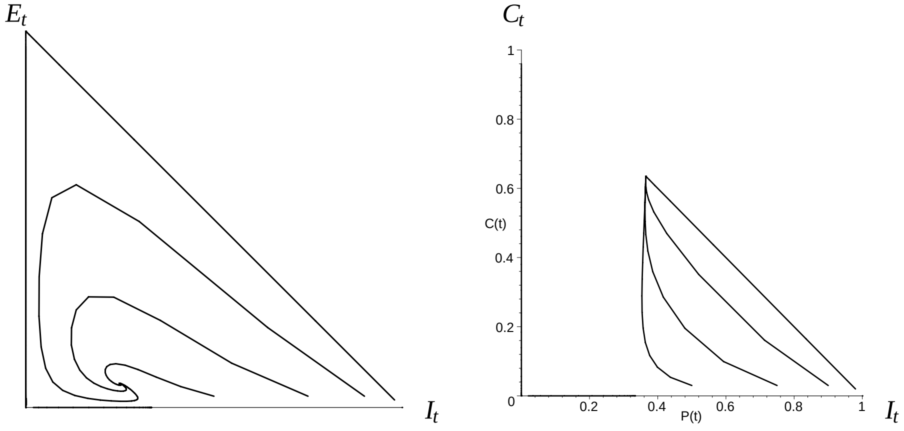
Figure 6: Expected payoffs for I and R as a function of I_t in the Innovator–Routinist Game

and thus $R_t^* = 1 - I_t^* = 2/3$. In other words, we find that equilibrium in this population game presupposes one third of Innovators and two thirds of Routinists. Since small perturbations of this equilibrium are corrected by the subsequent dynamics, we have reached an evolutionarily stable population profile (if only the two mentioned strategies are considered).

As soon as the method of evolutionary game theory is grasped, it is easy to study of the evolutionary dynamics of the Innovator–Routinist Game. However, other games can be much harder to analyse. It is, therefore, helpful to apply computer programs that, like Gambit (McKelvey et al., 2007), have a set of algorithms for calculating the Nash equilibria for complex games. When these equilibria are found, we can remove the uninteresting ones by means of Maynard Smith's definition of evolutionary stability, or by means of computer simulation.

The evolutionary dynamics of the Innovator–Routinist Game moves directly towards its evolutionarily stable Nash Equilibrium. In this way, it apparently contradicts the Schumpeterian vision of evolution as always taking place in waves of innovative activity punctuated by periods with predominantly routinist strategies. However, anyone who have studied dynamical systems knows that it is very easy to introduce cyclical movements of the state variables (here: the frequencies of the strategies). We simply need lagged responses. For instance, the preparation for the difficult switching from being a Routinist to being an Innovator might require a few periods. In the meantime, would-be Innovators continue to play as Routinists. This would lead still more Routinists to prepare for a switch. As a result, a wave of Innovators would emerge and it would continue to a point where the expected payoff of Innovators is clearly below average. Then the wave would turn towards the Routinist strategy, and so on. However, we shall ignore this possibility in the present paper.

Instead of considering repeated waves of behavioural change, the presented Schumpeterian games are more suited to study the compatibility of different strategies. More concretely, we study the effects of Exploiters who undermine the activity of Innovators and Complementors who promote this activity. Let us first take the Innovator–Routinist–Exploiter Game of Figure 4(a) and see how the relevant Nash Equilibrium is changed. The equilibrium of the game without Exploiters had one third of Innovators



(a) (b)

Figure 7: Dynamics of two Schumpeterian games. (a) Movement of I_t and E_t in the Innovator–Routinist–Exploiter Game. (b) Movement of I_t and C_t in the Innovator–Routinist–Complementor Game.

and two thirds of Routinists. At that equilibrium a player who mutates to an Exploiter will perform above the average $\pi(mix_{t'}) = 0$. It's payoff $\pi(E, mix_{t'}) = 7/3 - 4/3 = 1$. Therefore, the strategy will spread rapidly. However, this spread will imply a decrease of the frequency of exploitable innovators. In the end, the population profile will approach a Nash Equilibrium in which the frequency of Routinists have increased when compared with the equilibrium of the Innovator–Routinist Game. In contrast, the frequency of Innovators have decreased and Exploiters are present at a low frequency. This population profile of the Nash Equilibrium of Innovator–Routinist–Exploiter Game is $mix^* = (I_t^*, R_t^*, E_t^*) = (1/4, 11/16, 1/16)$.

Different trajectories to the stable equilibrium of the game are depicted in Figure 7(a). Here we exploit the fact that $I_t + R_t + E_t = 1$. Therefore, we only have to plot the frequencies of two of the strategies. Let us consider the trajectory that is initialised with I_0 close to 1, $R_0 = 0$, and a low frequency of Exploiters (the lower right corner of the graph). This trajectory moves to the upper right corner in which Innovators are brought to extinction. However, this equilibrium is not evolutionarily stable since the inclusion of Routinists at low frequency implies a totally different trajectory. This trajectory also starts in the lower right corner, but it ends in the evolutionary stable Nash Equilibrium. Here we see that $I_t^* = 1/4$ and $E_t^* = 1/16$. Therefore, we find $R_t^* = 1 - 1/4 - 1/16 = 11/16$.

The Innovator–Routinist–Complementor Game of Figure 4(a) provides of with a very different dynamics. The reason is that Complementors support the activity of Innovators. Thereby no room for Routinists are left, and they are brought to extinction. Let us, like above, start at the equilibrium of the Innovator–Routinist Game. Here a player who mutates to Complementor will obtain $\pi(C, mix_t) = 4/3$ compared with the average of zero. Therefore, the strategy will spread and this implies an improved performance of the Innovators. The population profile of the Nash Equilibrium is $mix^* = (I_t^*, R_t^*, C_t^*) = (23/63, 0, 40/63)$. The trajectories to this stable equilibrium are depicted in Figure 7(b). The trajectory that is initialised the lower right corner of the graph has I_0 close to 1, $R_0 = 0$, and a low frequency of Complementors. This trajectory moves to the the mentioned equilibrium the frequency of Complementors are larger than that of Innovators and where no Routinists exists. Any other trajectory end with the same the population profile. The only exceptions are the evolutionarily uninteresting equilibria in which only one strategy is represented in the population.

Let us end by shortly considering the Innovator–Routinist–Mixer Game 5. In this game there are players performing the pure strategies of Innovators and Routinists as well as players that for each game makes a choice of being an Innovator with a certain probability; if this strategy is not chosen, they play Routinist. In Figure 6 we see the scope for improving the payoff by the Mixer strategy. According to classical game theory the figure shows that, for instance, no rational player will be a Routinist in a population dominated by Routinists (i.e. where I_t is close to zero). Any Mixer strategy will do better. However, this strategy does not change the basic structure of the game. The problem is that any Mixer strategy will perform worse than the Innovator strategy in that situation. Similarly, the any Mixer strategy will perform worse than pure Innovators when Routinists predominate. Therefore, the Nash Equilibrium includes all three strategies.

6. Conclusion

The purpose of the present paper was to demonstrate the truth of Schumpeter's (1954b, 39) statement that "a new apparatus poses and solves problems for which the older authors could hardly have found answers even if they had been aware of them". This statement is not least true for Schumpeter's magnificent vision of capitalism as an evolutionary process of creative destruction. He simply lacked an analytical apparatus appropriate for exploiting his vision in an efficient manner. In combination, Nelson and Winter's *An Evolutionary Theory of Economic Change* and Maynard Smith's *Evolution and the Theory of Games* help to overcome this problem. For the study of a fully specified process of Schumpeterian competition, the Nelson–Winter model still seems to provide one of the best available set of tools. However, even extended forms of this model seriously constrain the treatment of different aspects of the Schumpeterian vision. Therefore, evolutionary game theory was suggested as a core complement. To prepare for this complement, the Nelson–Winter model was presented in terms of the equations of evolutionary dynamics that are important within evolutionary game theory. Then it was shown that the set of Schumpeterian strategies can be extended from innovators and imitators to routinists, complementors, and mixers. These strategies were presented in a modified version of John Maynard Smith's Hawk–Dove Game. In this setting, the possibility of coexistence of several of these Schumpeterian strategies was proved.

Evolutionary game theory has not followed Maynard Smith's focus on the Hawk–Dove Game. Therefore, one might ask for alternative starting point within game theory for studying Schumpeterian strategies. Such an alternative is hardly found in the large and conspicuous literature on the evolution of cooperative behaviour. The repeated Prisoner's Dilemma that ranges from Axelrod (1984) to Nowak (2006, Chs 5, 7, and 9) is simply not reflecting Schumpeter's vision of creative destruction. However, we might have considered the Ultimatum Game. This game between proposers and responders has been studied extensively, both theoretically and experimentally. We might think of a radical Schumpeterian innovator whose project is expected to give raise to a significant payoff. However, the innovator can only carry through the project with the help of other agents. To induce this support, they are proposed a share of the payoff. The question is which share will lead to the necessary help from the responder. This is especially important for securing credit for the carrying through of the radical innovative project in a relatively routinised environment. Here Schumpeter considered credit as a major constraint.

Classical game theory has a simple solution to the Ultimatum Game. A rational responder should accept any share of the proposer's expected payoff. Therefore, a rational proposer should offer a share close to zero. This solution is also obtained if the game dynamics is modelled by the replicator equation. However, evidence from many countries demonstrate that this is not the outcome of simplified experiments with sure money. On the contrary, the responder normally rejects offers that are considered "unfair" (Henrich et al., 2001). This is probably due to the fact that the accepted share of the payoff has an influence on the offers that the responder will receive in similar games in the future (Nowak et al., 2000). The reputation of demanding a fair share will thus influence the long-term payoff of the player. The resulting evolutionary dynamics often leads to shares that are not far from a fifty–fifty split, both in experiments and in terms of replicator dynamics. We might consider the Schumpeterian Ultimatum Game in the same vein. The finance of a radical innovation has per definition no customary sharing rule. Since the share accepted by the responder nevertheless may influence its reputation in the predominant routine-based games, it seems likely that it is very difficult for the innovative proposer to define an acceptable sharing. Even the early imitators may have problems. However, rules of sharing will gradually emerge, and this eases the situation for late-coming imitators. By taking this process into account, it becomes easier to understand an important aspect of Schumpeter's vision. It also becomes clear why this vision is of more importance for immature economic systems that in situations where innovation and imitation have become regular aspects of economic life.

Although the Ultimatum Game can hardly serve as an alternative to the suggested Schumpeterian versions of the Hawk–Dove Game, it does emphasise two important issues. First, the formalisation of the Schumpeterian vision must take place at many levels, and one of them is related to basic characteristics of human behaviour. Second, evolutionary game theory is closely related to experimental economics and other empirical studies. Any successful development of Schumpeterian games have to follow this lead.

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