

**From Schumpeter's failed econometrics to  
modern evometric analysis:  
Creative destruction as a tale of two effects**

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Version: 14 May 2004

Paper for the Conference of the International Schumpeter Society  
Bocconi University, Milan, 9–12 June 2004

## 1. Introduction

Although the concept of creative destruction clearly reflects core elements of Schumpeter's vision of capitalist economic evolution, he never developed its implications in a systematic manner. The reason for this neglect is, according to the present paper, that Schumpeter lacked evometrics—i.e. a toolbox for measuring and analysing evolutionary change that is derived from a general understanding of evolution (Metcalf, 2002; Andersen, 2003; 2004a; 2004b).<sup>1</sup> Such a toolbox is presently emerging, and the paper demonstrates that it immediately helps to explore creative destruction and other Schumpeterian concepts. It may also help to systematise the quantitative methods that have been developed in relation to applied evolutionary economics.<sup>2</sup>

A major application of the suggested evometrics is to depict the process of creative destruction as the history of two effects: the selection effect and the innovation effect. The selection effect covers what economic evolution has in common with biological evolution. This effect has a negative part—destruction—that reflects that firms and other economic entities with subnormal characteristics are shrinking and disappearing. It also has a positive part—creation—that reflects that firms and other economic entities with supernormal characteristics are promoted. We are thus facing Darwinian evolution in the case where the selection effect is predominant. But economic evolution is also characterised by a broadly defined innovation effect that includes elements of Lamarckian evolution. More specifically, the 'innovation' effect includes innovation in the narrow sense as well as imitation and learning. While the former clearly function as a part of Darwinian economic evolution (by providing new fuel for the selection process), this is not its only function. Innovation in the narrow sense also functions as an insurance against destruction for many firms. When it comes to imitation and learning, this motive of avoiding destruction is even more predominant. Therefore, the question of the relative strength of the two effects is of crucial importance for the characterisation of the process of creative destruction. In some cases we may see a process dominated by the selection effect that serves to promote small entrepreneurial firms, but in other cases we may see a process that is totally dominated by the innovation effect (fuelled by a threat of destruction for unchanged firms).

Since the evometric toolbox underlying the paper's analysis of the tale of creative destruction is presently rather unknown, the exposition progresses in a roundabout manner. Section 2 shortly presents Schumpeter's concept of creative destruction, his failed attempts to apply econometrics to his analysis, and the emergence of a general evometrics in relation to biometrics. Section 3 describes in a rather technical way (and supported by an appendix) how modern evometric tools can be applied to the analysis of the process of creative destruction. Section 4 returns to the Schumpeterian process of creative destruction to rethink it as a tale of two evometrically defined effects. Finally, section 5 makes a summary and draws some conclusions.

## 2. Evometrics as a missing link in Schumpeter's analysis

Many will agree with Nelson and Winter's (1982, 39) statement that 'we are evolutionary theorists *for the sake* of being neo-Schumpeterians—that is, because evolutionary ideas provide a workable approach to the problem of elaborating the Schumpeterian view of capitalism as an engine of progressive change.' However, although modern evolutionary theorising has during the last couple of decades promoted a kind of Schumpeterian renaissance, it is still true 'that this renaissance has, so far, been an excessively partial one. That is, it has confined itself to a rather restricted portion of a much larger body of thought' (Rosenberg, 1986, 197). We are still far from providing *A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*, which is the programmatic subtitle of Schumpeter's (1939) voluminous *Business Cycles*. In this book Schumpeter tried to combine the theoretical analysis of the process of capitalist evolution with the historical and statistical analysis of the facts of that process, but he largely failed (Kuznets, 1940). In this respect the Schumpeterian research programme still remains an unresolved challenge for evolutionary economists. Let us consider this challenge in relation to his concept of creative destruction.

### 2.1. Schumpeter's vision of creative destruction

Schumpeter's most condensed characterisation of 'the capitalist process' is found in his phrase of 'creative destruction'. This catchy phrase is a piece of thought-provoking rhetoric that clearly expresses his vision of capitalism:

The essential point to grasp is that in dealing with capitalism we are dealing with an

evolutionary process. ... [It is a process] that incessantly revolutionizes the economic structure *from within*, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism. It is what capitalism consists in and what every capitalist concern has got to live in. (Schumpeter, 1950, 82 f.)

Through the concept of creative destruction Schumpeter effectively pushes aside standard ideas about economic change. First of all, economic evolution is not a simple growth process in which all sectors of economic life expand in a balanced way. Instead it is characterised by the creation of novelty and the destruction of old products and processes. Furthermore, the existing firms and other organisations do not smoothly upgrade their competencies and switch their areas of specialisation. Instead they often perish in the evolutionary process. Finally, employees that lose their jobs are often facing great stress and significant welfare losses that seem more obvious than their long-term advantages of capitalist evolution. Their reactions constitute a permanent challenge to the institutions of capitalism. Thus creative destruction is a concept that reflects the competitive struggle and that emphasises the reactions to the temporary welfare costs.

The Schumpeterian challenge is to transform this verbally described vision into an operational analysis of actual processes of economic evolution. This operationalisation requires three complementary efforts. First, we need to develop a theoretical and statistical analysis of the *creation* of new structures and the *creative destruction* of old structures. Second, we need to analyse the changing institutional framework that determines the conditions for the creation and destruction of economic structures. Third, we have to deal with Schumpeter's (1939, 193 ff.) insistence on a 'cyclical process of evolution'. Thus he in a footnote to the previous quotation adds:

Those revolutions are not strictly incessant: they occur in discrete rushes which are separated from each other by spans of comparative quiet. The process as a whole works incessantly however, in the sense that there always is either revolution or absorption of the results of revolution, both together forming what are known as business cycles. (Schumpeter, 1950, 83)

This vision of economic evolution as a cyclical process with upswings and recessions gives yet another critique standard view of economic life: the short-term benefits of overcoming of business cycles have long-term costs in terms of a slower process of evolutionary change. But the vision of cyclical evolution is difficult to specify, so it also represents a major challenge to present-day evolutionary economists.

## 2.2. Schumpeter's troubled relationship to econometrics

Schumpeter's own attempts of making operational his broadly conceived vision of creative destruction were largely concentrated in the 1930s, and they were closely related to the emergence of econometrics. The first issue of *Econometrica*—the journal of the Econometric Society—came out in the beginning of 1933, and among its contributors were Frisch, Schumpeter and Tinbergen. Both the society and its journal suggested an abstract research programme that was most clearly expressed in the editorial by Frisch (1933) and the programmatic article on 'The Common Sense of Econometrics' by Schumpeter (1933). The programmatic task of econometrics was to overcome the tendency toward dichotomic specialisation among economists into theory without measurement and measurement without theory. This task was to be accomplished by establishing what Schumpeter (1954, 1141) later called 'the alliance between statistics and theoretical economics'. Somewhat more precisely, Frisch (1933, 2) proposed that the 'mutual penetration of quantitative economic theory and statistical observation is the essence of econometrics'. Several of the young econometricians (including Frisch, Leontieff and Tintner) were eager to help Schumpeter in transforming his theory into a quantitative format that would make it statistically operational, but only few and limited results were obtained.

Schumpeter's critical evaluation of the work of the econometricians was made when he in November 1949 addressed a conference on empirically oriented studies of business cycles. This conference was attended by a large part of the econometrically inclined economists, and to this audience he chose to make a provocation:

To let the murder out and to start my final thesis, what is really required is a large collection of industrial and locational monographs all drawn up according to the same plan and giving proper attention on the one hand to the incessant change in production and consumption functions and on the other hand to the quality and behavior of the leading personnel. (Schumpeter, 1949, 328)

Through the suggested studies Schumpeter (1949, 329) wanted to overcome what he considered to be 'the most serious shortcoming of modern business-cycle studies', namely 'that nobody seems to understand or even to care precisely how industries and individual firms raise and fall and how their raise and fall affects the aggregates'. By engaging in studies of the real process of creative destruction this abstraction would be revealed as unwarranted, and much econometric work on unanalysed aggregates would have to be rethought or scrapped.

Since Schumpeter died a couple of months after making his statements about the ‘historical approach to business cycles’, we shall never know exactly how he intended to implement them. But it is obvious that they brought an uncomfortable feeling to—and seriously shocked—many of his econometrically oriented friends and colleagues, for he seemed to suggest that they should drop most of their achievements and, more or less, start from scratch. Given his important role in the establishment of the econometric research programme, this seemed to be a strange change of mind. Tinbergen’s (1951, 59, 61) judgement was that although Schumpeter was able to express ‘warm sympathy’ for econometrics, he ‘lived another life’ and was ‘to some extent alien to’ econometric work.<sup>3</sup> In contrast, Frisch (1951, 9–10) judged that Schumpeter had the ‘intuition’ of what to model that is ‘the vital part of our science and *the true criterion of an econometrician*’, and he utilised econometric tools wherever possible. Thus, Schumpeter’s lacking use of the established econometric tools should be seen as a challenge to develop new ones that are better suited for implementing his vision of creative destruction.

Schumpeter’s ‘econometric’ attempts are recorded in his *Business Cycles*. Here he (Schumpeter, 1939, chs 5, 8–13) tried to handle the statistical facts of creative destruction, but he obviously got stuck into aggregative analysis. In retrospect, this is hardly surprising. Since Schumpeter was fond of architectural analogies, I shall phrase the problem in terms of econometric researchers working at different floors of a large house.<sup>4</sup> At the top floor we have the macroeconometricians with a language adapted to their main data and main problems. In the basement we have the microeconometricians, the researchers dealing in their own terms with the data and problems on firms, industries and regions. In the more or less empty intermediate floors there were plenty of room for researchers for trying to link between the two kinds of research. It was exactly this linking that Schumpeter was engaged in, but he found no relevant tools for supporting his kind of research. Instead his linking attempts made him intensively aware of the deep problems of performing isolated macrostudies as well as isolated microstudies. But the main result of this awareness of analytical difficulties was that he to a large extent spoilt his efforts at both the top and the bottom. Furthermore, he argued as if verbal historical accounts for creative destruction could function as a substitute for microeconomic results and the linking of them to macrodynamics. In the present paper it shall, however, be argued that what Schumpeter implicitly called for was evolutionary metrics, or evometrics.

### 2.3. *Biometrics and econometrics*

While Schumpeter was struggling with his theoretical, statistical and historical analysis of economic evolution, the analogous analysis of the process of biological evolution made a large jump forward. The background for this jump should not least be found in the late nineteenth and the early twentieth century with the emergence of the discipline of biometrics (also known as biometry). Although both the name and the research programme of econometrics were clearly inspired by biometrics (see below), the two disciplines differed in a very fundamental way. While standard econometrics was dominated by a need of getting rid of the disturbing effects of economic evolution in order to simplify the study economic problems, biometrics was largely developed to measure and analyse biological evolution (Porter 1986; Provine 1971; Depew and Weber 1995). The core founders were Galton (Darwin's cousin), Pearson and Weldon, and the editorial article of the first issue of their journal *Biometrika* (from 1901) emphasised that Darwin's theory of evolution by natural selection had a statistical character and that its development needed a statistical examination of a large number of cases. The development of the biometric research (partly in relation to Mendelian genetics) led to breakthroughs in both statistical methods and evolutionary theory during the 1920s and 1930s—not least thanks to R. A. Fisher (1999). Thus the programme of biometrics appears from its very start to be comparable to the evolutionary econometrics that Schumpeter asked for rather than to standard econometrics. Furthermore, biometrics has provided many tools for theoretical and empirical analysis that appear to be more or less directly applicable to an evolutionary econometrics. But although Frisch in his naming of econometrics and first journal was leaning heavily to biometrics and first journal—to the extent that he considered *Econometrica* to be the 'sister' of *Biometrika* (Divisia 1953, 22–30; Bjerkholt 1995)—there was not even a hint about any inspiration from the evolutionary nature of biometrics. So this window of opportunity was quickly lost, and in the subsequent definitions of econometrics the core came to emphasise the general combination of mathematics, economics and statistics (Tintner 1953).

When Schumpeter in the beginning of the twentieth century learnt about biometrics (mainly through the works of Galton and Pearson), it was unfortunately not at all clear that it could give an account for long-term evolutionary change. For instance, Galton thought that the mechanisms of inheritance would quickly cause

above-average variants to ‘regress to the mean’, so that sudden and somewhat mysterious jumps away from the mean were needed to explain real evolutionary change. Furthermore, Pearson engaged in a fruitless battle against the Mendelians who explored the mutations that in the end showed up to be a necessary element in any realistic explanation of long-term evolution. It was not before 1930 that Fisher (1999) was able to give a systematic account for biological evolution in which natural selection worked on hereditary and mutating characteristics in a way to produce evolutionary change. Although Fisher’s account was inspired by statistical thermodynamics and as a consequence was developed in a very general way (Depew and Weber 1995), this generality was difficult to comprehend because of the inclusion of specific biological mechanisms. This meant that none of his contemporaries among economists recognised the applicability of his formalised evolutionary theory. For Schumpeter an additional problem was that Fisher was applying a gradualist approach that ran contrary to his saltationist approach. So even if the mature Schumpeter had been willing to give up his well-known aversion against borrowing from other disciplines, he would have gained little *immediate* help for his explanations of long-term economic evolution. However, a deeper look into Fisher’s and Schumpeter’s views of evolution demonstrates surprising similarities (Foster 2000).

Today it is not necessary to study the complex history of biometrics to grasp its importance for evolutionary economics. The reason is that researchers have transcended the biometric tradition to develop what may be called a general evolutionary metrics. Although there are several contributors to this generalisation, it was least that George R. Price was able to sum up and develop its main lessons in a few papers that were written in the beginning of the 1970s (Price 1970; 1972a; 1972b; 1995; Maynard Smith and Price 1973). Price was a rather strange person who late in life started his evolutionary work based on an interest in human affairs and as an entrant to biometric analysis (Hamilton 1996, 171–176, 318–327; Frank 1995), and these characteristics may help to explain why he developed what I call evolutionary metrics or evometrics.<sup>5</sup> His work helped to lay the foundation for evolutionary game theory, clarified Fisher’s main result about natural selection, developed a general and very fruitful partitioning of *any* evolutionary change, and helped to overcome the fruitless controversy on group-based selection versus individual selection (Frank 1995, 374–383). Of special interest is the simple and powerful evolutionary mathematics that Price developed to overcome some of the ambiguities of Fisher’s

analysis. His most conspicuous result is a partitioning of the evolutionary change that included not only the effect of selection but also the effect of causes that increase variation. Price's equation has a certain 'queerness' (Frank 1995, 376) that has to be understood in order to apply it efficiently. This effort has not least been made by Frank (1995; 1997; 1998), who has become a major contributor to the development and diffusion of Price's partitioning of evolutionary change. Frank demonstrates—in a 'highly original, almost idiosyncratic manner' (Nowak and Sigmund 1998, 760)—that a large number of evolutionary problems can be clarified by means of Price's equation. He has, furthermore, to a large extent upheld the generality of Price's work (cf. Price 1995). This means that we are not only facing a contribution to biometrics but also to general evometrics, which serves surprisingly well as a means of accounting for economic evolution and thus to explore creative destruction.<sup>6</sup>

In the following we shall see how such evometric accounts can be made. But from the very beginning it should be made clear that the biologists are dealing with Darwinian evolution in which the selection effect is predominant. In contrast, economic evolution is also characterised the fact that firms strive to become destructors rather than being destroyed. Thereby, a broadly defined innovation effect (including learning, imitation and innovation in the narrow sense) becomes important in evolutionary economics—although it includes elements of what in biology is considered defunct Lamarckian evolution. As a result we have to confront the question of the relative importance of 'Darwinian' and 'Lamarckian' elements of the process of creative destruction. In some cases we may, for instance, see a process dominated by the selection effect that promotes innovation-based entrepreneurial firms, but in other cases we may see a process that is totally dominated by the innovation effect (fuelled by a threat of destruction for unchanged firms).

### 3. Evometric partitioning the process of creative destruction

Creative destruction is a process in historical time. To study the switching of resources implied by this process we start by selecting a sequence of points of time,  $t, t', t'', \dots$ , and by defining a set of economically relevant entities for which we collect census data at these points of time. The time points, the set of entities (here called the 'population'), and the individual entities may pragmatically be defined in many ways. Thus the time period between our collections of census data may e.g. be one or ten

years, the ‘population’ may cover an industry, a country or the whole world, and the entities may be plants, firms or regions. In any case, the basic task is to describe the creative destruction that takes place between the population state  $P$  and the population state  $P'$ . This change is produced by the basic evolutionary forces of selection and innovation as well as by exogenous factors. Since we put some emphasis on selection, it is convenient to call  $P$  the pre selection population and  $P'$  the post selection population. For concreteness, we may think of  $P$  and  $P'$  as consisting of firms, and until section 4.1. we shall ignore the problem of handling new firms when collecting the necessary information for our analysis.

### *3.1. Partitioning of firm-based reproduction*

In biology the core concept of evometric analysis is denoted by ‘fitness’, and this naming is related to the phrase of ‘survival of the fittest’ that means something like the ‘evolutionary success of the best designed’ (Michod, 1999, 140). In the present version of economic evometrics the neutrality and non-theoretical character this concept are emphasised by using the term ‘reproduction’ rather than fitness. The degree of reproduction of a firm  $i$  is measured by its resources  $x'_i$  in the post selection population compared with its resources  $x_i$  in the pre selection population. Thus the core variable is the reproduction coefficient  $w_i = x'_i / x_i$ . Following Marx, the firm (and any other socio-economic entity) may be said to be characterised by ‘simple reproduction’ if  $w_i = 1$ , ‘expanded reproduction’ if  $w_i > 1$ , and ‘shrinking reproduction’ if  $w_i < 1$ . These terms form the basis of an analysis of creative destruction that even in its simplest form require a good deal of notation (summarised in table 1).

**Table 1: Notation for the basic measurement of creative destruction.**

Variable	Description	Definition
$X, X'$	variables for the pre selection population and the post selection population	
$x_i$	resources of entity $i$	
$x$	resources of total population	$\sum x_i$
$s_i$	entity $i$ 's resource share	$x_i / x$
$w_i$	reproduction coefficient of entity $i$	$x'_i / x_i$
$\Delta w_i$	change in entity $i$ 's reproduction coefficient	$w'_i - w_i$
$w$	mean reproduction coefficient	$\sum s_i w_i$
$\Delta w$	change in mean reproduction coefficient	$w' - w$
$\text{Var}(w_i)$	variance of reproduction coefficients	$\sum s_i (w_i - w)^2$
$E(w_i \Delta w_i)$	expected value of change in reproduction coefficients	$\sum s_i w_i \Delta w_i$

Notes: 'Population' means the set of entities rather than the population of potential employees. The non-standard subscripts in the notation for variance and expected value are included to allow for hierarchical partitioning (see section 3.2).

The fact that the reproduction coefficients of firms are normally different and influenced by their characteristics implies that a process of selection is taking place. In the study differential reproduction we often concentrate on relative reproduction coefficients  $w_i / w$ , where  $w$  is the population's (weighted) mean reproduction coefficient.<sup>7</sup> The relative reproduction coefficient of a firm is, of course, related to its resource share (e.g. its employment share). More specifically, the resource share in the post selection population  $s'_i = s_i w_i / w$ . Some firms increase their resource shares because they have characteristics that imply that they have above average reproduction while others have characteristics that imply that their resource shares decrease. The former are 'positively selected' while the latter are 'negatively selected'. Thus we are clearly facing a process of 'relative creative destruction'. We are, however, not necessarily facing 'absolute creative destruction'. If the employment of the overall population is growing, the firms that are negatively selected in relative terms might increase their absolute employment. But when considering creative destruction, we normally think in terms of the increase and decrease of the absolute employment of the firms. The reason is that it is the movement of employment away from weak firms that is most closely related to the social costs of economic evolution.

Let us now consider the reproduction coefficients as reflecting a combination of

intrinsic properties of the firms (e.g. their ‘normal’ productivities) and characteristics of their environment. Then we can describe the change of the in the mean reproduction coefficient  $\Delta w$  as the outcome of three effects: the selection of firms with different characteristics, the innovative change in these characteristics, and the change of the characteristics of the environment. The first effect is *selection effect* that may be defined as the relative variance of the reproduction coefficients ( $\text{Var}(w_i)/w$ ). According to this definition, selection is the evolutionary mechanism that assigns reproduction coefficients to the firms of the pre-selection population. The population-level effect of this firm-level selection is a change in the mean reproduction coefficient. There are, however, two additional reasons for this change. First, the environment of the population may have changed so that individual reproduction coefficients as well as the mean reproduction coefficient are different in the post selection period. Second, the individual firms may have performed (localised) innovations that influence their reproduction coefficients and the mean reproduction coefficient. Thus we have to add the combined effect of environmental change and innovation to the selection effect. For simplicity we shall call this combined effect the *innovation effect*—although environmental factors will often be its dominant determinant.<sup>8</sup> This innovation effect is defined as the relative expected value of the change of the reproduction coefficients ( $\text{E}(w_i\Delta w_i)/w$ ).

By means of these definitions the derivation of equation (A7) in the appendix demonstrates that that any kind of total evolutionary change can be partitioned in the following way (see also Frank, 1998, 13–15, 21):

$$T = \Delta w = \underbrace{\frac{\text{Var}(w_i)}{w}}_{\text{Selection effect}} + \underbrace{\frac{\text{E}(w_i\Delta w_i)}{w}}_{\text{Innovation and environment effect}} = S^F + I^F. \quad (1)$$

This equation (1) tells that the change in the mean reproduction coefficient can be partitioned into a ‘selection effect’ and an ‘innovation and environment effect’ (shortened to ‘innovation effect’). The selection effect is denoted  $S^F$  to indicate that it is defined with respect to a population composed of firms. It describes how the aggregate change is influenced by the differences in the reproduction coefficients of the first period.  $S^F$  is zero if all reproduction coefficients are equal. The innovation (and environment) effect  $I^F$  describes the outcome of the changes in the reproduction coefficients. It is zero if no reproduction coefficient undergoes any change. Although

equation (1) is describing evolutionary change between two periods, it gives some information of future conditions for evolution. This is most obvious if we are facing a pure selection process (i.e.  $I^F = I'^F = 0$ ). For such a process  $S'^F < S^F$  since  $w' > w$  and  $\text{Var}(w'_i) < \text{Var}(w_i)$ .

From equation (1) we may quickly move to the discussion of *relative* creative destruction. To do so we note that half of the selection effect must come from firms with decreasing employment shares. The question is how these firms perform with respect to the change in their reproduction coefficients. To study this question we start by splitting the firms into two subpopulations  $c$  and  $d$ . Firms are members of subpopulation  $c$  that consists of relatively growing ('creative') firms if  $w_i \geq w$ . Otherwise they are members of the subpopulation  $d$  that consists of firms that are encountering any degree of relative decline/destruction. Then we split up the firm-level selection effect and the firm-level innovation effect of equation (1) according to the membership of these subpopulations:

$$\begin{aligned}
 S^F &= \underbrace{\frac{\text{Var}(w_i)}{2w}}_{\text{Creative selection effect}} + \underbrace{\frac{\text{Var}(w_i)}{2w}}_{\text{Destructive selection effect}} = S_c^F + S_d^F, \\
 I^F &= \underbrace{\frac{\sum_{i \in c} s_i w_i \Delta w_i}{w}}_{\text{Innovation-response-to-increase effect}} + \underbrace{\frac{\sum_{i \in d} s_i w_i \Delta w_i}{w}}_{\text{Innovation-response-to-decrease effect}} = I_c^F + I_d^F.
 \end{aligned} \tag{2}$$

Let us consider equation (2) from the viewpoint of creative *destruction*. Here the important issue is whether or not the relative decline of firms in the first period is compensated by growth in the next period. The relative destructive selection effect  $S_d^F$  is a positive contribution to the total evolutionary effect. The problem is how relative destruction in the first period is connected to the innovation-response-to-decrease effect of the next period. If weakened firms did not react or reacted through a negative innovation effect, then they would face an irreversible process of decline. But they may also have the possibility of compensating reactions to their weakened position. Thus Schumpeter emphasises that the response of weakened firms is often to imitate the growing firms. The response is even more radical in Simon's (1982) theory of satisficing behaviour. According to this theory, super-normal performers are satisfied and do nothing to improve their performance. In contrast, sub-normal performers are dissatisfied and engage in innovation. Thereby, they may be able to improve their position. The crucial question for the long-term process of relative

creative destruction is, therefore, whether  $I_c^F < I_d^F$ . This is, of course, an empirical question, and it is not easy to answer. One of the difficulties is that random factors play a large role in determining growth rates, and therefore we see a large degree of ‘regression’ toward mean behaviour.

To include the issue of *absolute* creative destruction into our analysis, we only need to change slightly the above procedure. Firms are members of subpopulation  $C$  growing firms if  $w_i \geq 1$ . Otherwise they are members of the subpopulation  $D$  that absolutely declining. Then we split up the firm-level selection effect and the firm-level innovation effect of equation (1) according to the membership of these subpopulations:

$$\begin{aligned}
S^F &= \underbrace{\sum_{i \in C} s_i (w_i - w)^2}_W + \underbrace{\sum_{i \in D} s_i (w_i - w)^2}_W = S_C^F + S_D^F, \\
I^F &= \underbrace{\sum_{i \in C} s_i w_i \Delta w_i}_{\text{Innovation-response-to-growth effect}} + \underbrace{\sum_{i \in D} s_i w_i \Delta w_i}_{\text{Innovation-response-to decline effect}} = I_C^F + I_D^F.
\end{aligned} \tag{3}$$

Here any subeffect is zero if its subpopulation has no members.

The questions related to equation (3) are more or less the same as those related to equation (2). But now the problem of firing of employees enters into the discussion. In the long run the movement of employees from declining to growing firms determines the absolute destruction effect. But this movement is seldom a smooth one. From the viewpoint of the moving employees there is often a welfare loss due to temporary unemployment and loss of social context. From the viewpoint of the declining firms the problem is that the loss of employees may in many ways influence productivity in a negative way. Thus we encounter not only new reasons for compensating responses to decline but also new difficulties of avoiding vicious circles.

### 3.2. Partitioning of firm-and-plant-based reproduction

If firms are composed of plants, the innovation effect of equation (1) may be due to two different subeffects. First, the plants (denoted by  $ij$ ) within a given firm (denoted by  $i$ ) may have different reproduction coefficients in the first period. Second, the plants may change their reproduction coefficients between the first and the second period. Thus the ‘innovation’ effect measured at the level of firms may be partitioned into a firm-level selection effect and a plant-level innovation effect. This partitioning

is additive and it may immediately be included into equation (1):

$$T = \Delta w = \underbrace{\frac{\text{Var}(w_i)}{w}}_{\text{Firm-selection effect}} + \underbrace{\frac{\text{E}(\text{Var}(w_{ij}))}{w}}_{\text{Plant-selection effect}} + \underbrace{\frac{\text{E}(\text{E}(w_{ij}\Delta w_{ij}))}{w}}_{\text{Plant innovation effect}} = S^F + S^P + I^P. \quad (4)$$

This version of the partitioning of the change of the mean reproduction coefficient allows us to consider two levels of selection: On the one hand, there is a selection mechanism that works at the level of the overall population. This mechanism promotes and demotes the employment of firms. On the other hand, the firm has its internal selection mechanism that promotes and demotes the employment of plants. If the firm consists of a single plant or of plants that grow at the same rate, this effect is zero. But often this is not the case.

In the context of firm-and-plant-based growth there is a need of augmenting our previous analysis of the process of relative and absolute creative destruction. Let us concentrate on absolute creative destruction. Here the task is to define the relevant subpopulations of firms and plants. On this background we may partition the innovation effects of equation (3):

$$\begin{aligned} I_C^F &= S_C^P + I_{CC}^P + I_{CD}^P, \\ I_D^F &= S_D^P + I_{DC}^P + I_{DD}^P. \end{aligned} \quad (5)$$

Here each of the innovation effects within both increasing and decreasing firms is partitioned into a selection effect and two innovation effects. The plant-based selection effects ( $S_C^P$  and  $S_D^P$ ) reflect the composition of firms with respect to plants of supernormal and subnormal reproduction coefficients in the first period. The innovation effects ( $I_{CC}^P, I_{CD}^P, I_{DC}^P, I_{DD}^P$ ) represent the change of the reproduction coefficients of plants with positive and negative reproduction coefficients. The first letter of the subscripts of these innovation effects refers to the firm while the second letter refers to the plant.

Let us briefly consider the issues of absolute creative *destruction* suggested by these definitions. The possibility of declining firms to react by promoting relatively highly performing plants ( $S_D^P$ ) depends on the variance of their plants with respect to growth potential. Over time this possibility will become exhausted unless new variance is introduced. It is, therefore, important how the plants of decreasing firms responds. Here we have to types of responses: the response of their growing plants ( $I_{DC}^P$ ) and the response of their declining plants ( $I_{DD}^P$ ). In both cases we may

encounter the same responses as discussed with respect to the reaction of firms: imitation, satisficing behaviour and vicious circles. Thus it becomes obvious that the analysis of creative destruction is very complex. At the same time we come much closer to the realities of economic life than when applying decompositions that are directly imported from evolutionary biology. The crucial difference is that innovation effects are much more important and complex in economic evolution than in biological evolution.

### 3.3. *Reproduction functions and the partitioning of mean productivity change*

In the previous sections we have only studied creative destruction in terms of the reproduction of firms and plants, and this analysis was based on the assumption that the reproduction coefficients have an evolutionary meaning. This evolutionary meaning can be made explicit by relating the analysis to evolving characteristics that influence these coefficients. This can be done in terms of reproduction functions (in biology called fitness functions<sup>9</sup>) that are obtained by estimating the strength of selection by regressing reproduction coefficients on characteristics. Such reproduction functions may also be called selection functions, and they reflect an important aspect of the process of creative destruction. For simplicity we shall in the present paper only study the case where selection is based on a single characteristic, but it is possible to extend the analysis to multiple regression.

Since we cannot be sure that reproduction functions are straight lines, it is helpful to think in terms of polynomial regression:

$$w_i = \alpha + \beta(w_i, z_i)z_i + \gamma(w_i, z_i)z_i^2 + \varepsilon. \quad (6)$$

The polynomial in equation (6) describes the reproduction coefficient of a firm as influenced by its characteristic value  $z_i$  as well as by a deviation term  $\varepsilon$  (with a mean of zero). This influence is described by two coefficients.  $\beta(w_i, z_i)$  describes the slope of the reproduction function and  $\gamma(w_i, z_i)$  describes the degree with which the slope changes with increasing  $z_i$ . In the present paper emphasis is put on the case where  $\beta(w_i, z_i) > 0$  and  $\gamma(w_i, z_i) = 0$ . In this case we have *positive directional selection*. But for some characteristics we have that  $\beta(w_i, z_i) = 0$  and  $\gamma(w_i, z_i) \neq 0$ . If  $\gamma < 0$ , then the reproduction function has a maximum toward which the selection pressure is moving the firms (*stabilising selection*). Thereby the population keeps its mean but

decreases its variance. If  $\gamma > 0$ , then the reproduction function has an intermediate minimum from which the selection pressure is moving the firms away (*disruptive selection*). Thereby the population increases its variance, and in the end it may split up into two populations. More complex reproduction functions are characterised by other combinations of the coefficients of the quadratic polynomial or by higher-order polynomials.

In the following we shall only consider selection that takes the form of a linear and positively sloped reproduction function. Here the more general form of equation (6) largely serves as a caveat: we cannot be sure that real selection is that simple. However, the linear case might be found when the selection criterion is the productivity of firms (or plants)—especially if we manipulate the selection criterion to obtain the linear form (e.g. by considering the logarithm of the productivities). Even if the ignored selection criteria (like access to financial resources) and the random elements in the process of selection imply rather large deviations of individual observations from the reproduction function, the selection process will transform variance of productivities into change of mean productivity. Therefore, we may start from a naïve picture of the evolutionary process. But this is only a start. The reproduction function is, for instance, describing the ‘performance’ of firms with respect to employment, but in the end we have to confront the obvious fact that firms are not normally competing to maximise employment. There are many mediating links between their normal productivity and their change in employment (Metcalf, 1997). Thus the employment depends on the selection pressure from consumer search as well as on the prices set by firms. These prices depend on the price-setting power of firms as well as on their costs of production. These costs of production depend on the bargaining power of potential and actual employees as well as on the normal productivity of the firm. Since all these links are important and vary across industries and over time, it is obvious that there is no simple selection of firms with respect to their normal productivities. But it is important to note that the evolution with respect to productivities does not presuppose a high efficiency of the selection process. Even a fairly weak covariance between the reproduction coefficients of firms and their normal productivities may be sufficient to influence the long-term direction of the evolutionary process.

**Table 2: Additional notation for measuring the evolution of characteristics.**

Variable	Description	Definition
$f_i$	relative reproduction coefficient	$w_i / w$
$z_i$	value of characteristic of entity $i$	
$\Delta z_i$	change in value of characteristic of entity $i$	$z'_i - z_i$
$z$	mean value of characteristic	$\sum s_i z_i$
$\Delta z$	change in the mean characteristic	$z' - z$
$\text{Var}(z_i)$	variance of characteristics	$\sum s_i (z_i - z)^2$
$\text{Cov}(w_i, z_i)$	covariance of reproduction coefficients and characteristics	$\sum s_i (w_i - w)(z_i - z)$
$\beta(w_i, z_i)$	regression of reproduction coefficients on characteristics	$\text{Cov}(w_i, z_i) / \text{Var}(z_i)$
$E(w_i \Delta z_i)$	expected value of change in characteristics	$\sum s_i w_i \Delta z_i$

See also table 1.

When studying the evolving characteristic directly, we may define total evolutionary change with respect to a particular characteristic of a population as the change in the mean of the individual values of that characteristic  $\Delta z$ . Thus there is no evolutionary change if  $\Delta z = 0$ .<sup>10</sup> The calculations related to the analysis of this form of evolution is described in table 2, where we for convenience also have named the relative reproduction coefficient as  $f_i$ . For concreteness we shall take the productivities of firms ( $z_i$ ) as our example, but we shall ignore the difficulties of productivity studies. Instead we note that according to the definition there is no evolution in the unlikely case where there is no change in mean productivity but instead a cancelling out of positive and negative changes at the level of firms.

Given that we observe evolutionary change (i.e.  $\Delta z \neq 0$ ), the task is to partition it into a selection effect and an innovation effect. This partitioning is like that of equation (1), but we need a more general definition of the two effects. The *selection effect* is also called the selection differential, i.e. the part of the change in mean productivity (or any other characteristic) that is due to selection of firms (or plants) with *given* productivities. To determine the size of this differential we of course need to know the mean productivity in the pre selection population and rewrite is for later use:

$$z \equiv E(z_i) = E(f_i) E(z_i).$$

The validity of this rewrite is due to the fact that  $E(f_i) = \sum s_i w_i / w = 1$ . We also need

to consider the level of the mean productivity in the post selection population that is obtained by selection:

$$z'^S = \sum s'_i z_i = \sum s_i f_i z_i = E(f_i z_i).$$

Now we see that the selection differential

$$S^F = \Delta z^S = z'^S - z = E(f_i z_i) - E(f_i)E(z_i) = \text{Cov}(f_i, z_i) = \frac{\text{Cov}(w_i, z_i)}{w}. \quad (7)$$

This is Price's interpretation of the selection effect (with respect to firms), and this interpretation holds for the case of directional selection. It describes selection as the component of the evolutionary process that assigns reproduction coefficients to the firms of the pre selection population based on their productivities. For each member, selection determines the relative reproduction coefficient  $f_i = w_i / w$  that corresponds to its productivity  $z_i$ . If there are differences with respect to productivities, then the post selection population has evolved to the degree that the initial differences are exploited by selection.

The *innovation effect* is present if the individual productivities are not constant (as it was assumed in equation (7)). This innovation effect is simply the mean value of these changes measured in the post selection population. According to this definition, the innovation effect is determined by the weighted influence of the degree to which the firms of the post selection population have changed their productivities when compared to the pre selection population,  $\sum s'_i \Delta z_i$ . This definition may be rewritten:

$$I^F = \Delta z^I = \sum s'_i \Delta z_i = \sum s_i f_i \Delta z_i = E(f_i \Delta z_i) = \frac{E(w_i \Delta z_i)}{w}. \quad (8)$$

In the definition of the innovation effect used in equation (8) we are obviously applying another concept of innovation that the one used in neo-Schumpeterian innovation studies. While innovation in these studies is seen as the introduction of a positively evaluated novelty with respect to the overall population, we presently apply a neutral concept of innovation that covers any kind of local-level change. It simply means that something new has occurred with respect to firm-level productivities. Thus the productivity of individual firms may have increased or decreased. There are, of course, many potential reasons for both negative and positive values, but let us concentrate of the knowledge issue. In this respect productivity change may be positive because of innovation, imitation or learning processes. It might be negative because the firm does not have an effective system of reproduction of its knowledge.

The question is now how the selection effect and the innovation effect of equations (7) and (8) relate to the total evolutionary change  $\Delta z$  that we observe. Fortunately, this relationship is straightforward. The derivation of equation (A6) in the appendix demonstrates that total evolutionary change is simply the sum of the two effects:

$$T = \Delta z = S^F + I^F = \underbrace{\frac{\text{Cov}(w_i, z_i)}{w}}_{\text{Selection effect}} + \underbrace{\frac{\text{E}(w_i \Delta z_i)}{w}}_{\text{Innovation effect}}. \quad (9)$$

This is Price's equation (identity) that can be derived for any evolutionary process in biological life, economic life, and elsewhere (Price, 1995). Although it has more empirical contents than the previous partitioning of the change in the mean reproduction coefficient, it can be partitioned in the same way as before.<sup>11</sup> Thus we may include the fact that firms consist of plants to see that

$$T = \Delta z = \underbrace{\frac{\text{Cov}(w_i, z_i)}{w}}_{\text{Firm-selection effect}} + \underbrace{\frac{\text{E}(\text{Cov}(w_{ij}, z_{ij}))}{w}}_{\text{Plant-selection effect}} + \underbrace{\frac{\text{E}(\text{E}(w_{ij} \Delta z_{ij}))}{w}}_{\text{Plant innovation effect}} = S^F + S^P + I^P. \quad (10)$$

We may also make further partitioning of equations (9) and (10) to study absolute creative destruction like we did in equations (3) and (5).

To discuss the results it is useful to note that the present selection effects in terms of covariances can be expressed as the products of regression coefficients and variances. For instance,

$$\frac{\text{Cov}(w_i, z_i)}{w} = \frac{\beta(w_i, z_i)}{w} \text{Var}(z_i).$$

Here the variance  $\text{Var}(z_i)$  can be interpreted as the fuel of the selection process. Then the relative regression of reproduction coefficients on productivities ( $\beta(w_i, z_i)/w$ ) can be interpreted as the efficiency with which this fuel is exploited to bring about aggregate productivity change. A similar interpretation can be made for the plant-selection effect.

When we study empirical processes of creative destruction, it is not only productivity that evolves. But this fact is no hindrance for performing the type of analysis that has just been defined. If we study the process of creative destruction in terms of productivities, we normally make the hypothesis that some part of the actual change in the distribution of employments across firms (and plants) is due to the selection of firms according to their productivities. Thus we make the double assumption that there are differences with respect to productivity and that the complex

selection mechanism is strong enough to exploit these differences to bring about increase in aggregate productivity. We may also safely make the hypothesis that some part of aggregate productivity change is due to innovative productivity changes within individual firms. But in general we can say little of the relative strength of these two aspects of the evolutionary process. This is an issue for empirical analysis.

#### 4. Evometric exploration of Schumpeter's processes of creative destruction

##### *4.1. Relating to empirical studies based on longitudinal microdata*

The research in evolutionary economics in general and creative destruction in particular has to some extent been hindered by a lacking emphasis on the development of descriptive statistics that are relevant for theoretical, statistical and historical analyses of evolution. For instance, Nelson's (1981) survey of productivity studies emphasises the Schumpeterian ideas of heterogeneity and institutional creative destruction, but it does not apply the methods for decomposition implicit in his and Winter's theoretical work (e.g. Nelson and Winter, 1982, 165–175). The reason for this omission is probably to some extent that longitudinal microdata were missing at the time he was writing, but it is hardly necessary that we had to wait 20 years before another survey could conclude in a way that 'echoes Nelson's ... earlier analysis' and emphasises that 'it can now be addressed better quantitatively' (Bartelsman and Doms 2000, 591). To the extent that we are able to describe evolutionary processes by means of quantitative statistics and phrase our hypotheses in terms of these statistics, the gap between the evolutionary theory of creative destruction and the quantitative analysis can be narrowed down. Since recent research has provided a rather rich literature with some relationship to the issues of creative destruction, the most immediate task is to demonstrate that Price's equations (9) and (10) may be found by slight rewrites of a many formulas of applied economics.

From the new wave of microeconomic studies based on longitudinal data, we shall take the mentioned survey of productivity studies by Bartelsman and Doms (2000, p. 583; see also Foster et al. 1998). They emphasise a partitioning of aggregate productivity change that serves as 'a framework to interpret the seemingly disparate findings in the literature'. The core part of this partitioning refers to the decomposition of productivity change in the set of continuing plants (i.e. plants that exist at both  $t$  and  $t'$ ).<sup>12</sup> They (or rather Foster et al., 1998, 16) decompose aggregate

productivity change from the continuing plants in three components:

$$\begin{aligned}
\Delta z &= \underbrace{\sum \Delta s_i (z_i - z)}_{\text{Selection effect}} + \underbrace{\sum s_i \Delta z_i}_{\text{Within-plant effect 1}} + \underbrace{\sum \Delta s_i \Delta z_i}_{\text{Within-plant effect 2}} \\
&= \underbrace{\frac{\text{Cov}(w_i, z_i)}{W}}_{\text{Selection effect}} + \underbrace{\frac{E(w_i \Delta z_i)}{W}}_{\text{Innovation effect}} = S^P + I^P.
\end{aligned} \tag{11}$$

The first line in equation (11) is Bartelsman and Doms' preferred decomposition. From the derivations in the appendix it is easy to see that the first component of equation (11) is the selection effect of Price's equation (9). They call it the 'between-plant effect'. The second and third components combine to form the innovation effect, but Bartelsman and Doms argue to keep them distinct. Their within-plant effect 1 ( $\sum s_i \Delta z_i$ ) is called the 'within-plant effect' while they misleadingly call the within-plant effect 2 ( $\sum \Delta s_i \Delta z_i$ ) 'a covariance term'—a better name is the cross effect.

Although we may quickly derive Price's equation in quite diverse contexts, it should be emphasised that the data has not normally been handled according to the logic of evolutionary partitioning. This is clear from Bartelsman and Doms' work, and the consequence is that the partitioning in equation (11) is not really fully reflecting their work. To be more specific, equation (11)'s equalisation of the Bartelsman–Doms partitioning with Price's decomposition only holds if there are no entering plants in the industry. However, their partitioning includes components for both continuing plants, entering plants and exiting plants. If we denote the set of continuing plants by  $I$ , the set of entering plants by  $J$ , and the set of exiting plants by  $K$ , we may represent their partitioning of aggregate productivity change in the following way:

$$\Delta z = \underbrace{\sum_{i \in I} \Delta s_i (z_i - z)}_{\text{Incumbent selection effect}} + \underbrace{\sum_{i \in I} (s_i + \Delta s_i) \Delta z_i}_{\text{Incumbent innovation effect}} + \underbrace{\sum_{i \in J} s_i (z'_i - z)}_{\text{Entry effect}} - \underbrace{\sum_{i \in K} s_i (z_i - z)}_{\text{Exit effect}}. \tag{12}$$

The two added components in equation (12) are called the entry effect and the exit effect. They are very handy, but their inclusion means that the logic behind Price's equation is lost. The main reason is that the mean productivity is calculated for all firms. Therefore, there is no elegant interpretation of the individual components as covariances or expected values. An apparent solution would be to include the new effects in the old ones. This works for the exit effect, which can easily be included in the selection effects since in this case  $\Delta s_i = -s_i$  (and my name 'incumbent selection effect' really suggests such an inclusion). However, the entry effect cannot be included. The reason is that we cannot define reproduction coefficients for entering

firms (where we would have to put a zero in the denominator).

The problem of handling entering firms cannot be ignored in the analysis of the process of creative destruction. Both Schumpeter and many of his present-day followers in endogenous growth theory and industrial organisation are emphasising that this process is kept alive by the introduction of innovations by new firms that ultimately displace incumbent firms. Some researchers (like Carroll and Hannan, 1999) even suggest that evolutionary analysis can largely be performed in terms of a demography of firms that concentrates on ‘vital statistics’, i.e. the entry and exit events. This suggestion represents no fundamental problem for evometrics as long as any entrant can be connected to a member of the incumbent population. But the logic of selection and innovation that has been described in the present paper cannot work without the specification of such relationships (Price, 1995).

There are two complementary solutions to the problem of including new firms into evometric analysis. The first solution is to accept Schumpeter’s idea that they represent genuine breaks with the past. In this case we have to handle new firms in an ad hoc manner like in equation (12). This means that the elegant parts of evometrics can only be applied to the incumbent population of firms. Since such a population may contain a mix of relatively new and relatively old firms, we are still able to most of the real-life processes of creative destruction. But we lack a full treatment of the process. The second solution is to consider entrants as representing less novelty than assumed by Schumpeter. The most obvious case is when new firms can be considered as spin-offs of old firms. In this case the employment of a spin-off is included in the calculation of the reproduction coefficient of its mother firm. To allow for this inclusion we often need quite detailed information on the founding of firms (Klepper 2001). In practice, the two solutions to the problem of handling new firms should be applied in parallel, but there is a need of recognising that the mixing of different logics is likely to create some confusion for evolutionary interpretations of available microstudies and for the general analysis of creative destruction.

#### *4.2. Schumpeter’s cycles in evometric retrospect*

Schumpeter’s formulation of his concept of creative destruction is connected to his simplified scheme of evolutionary analysis, which represented the level of complexity that he felt he could safely handle. According to this scheme the evolution of the routine economy tends to take place through the following sequence of events:

- Initial equilibrium: The analytical starting point is an economic system that is based on solid routine behaviour. This system is assumed to have found an equilibrium that allows the economic agents year after year to operate in their accustomed ways.
- Innovation: The initial equilibrium breaks down when a wave of innovators gain access to some of the resources of the economic system. This leads to an economic upswing, but gradually the stream of innovations fades out because of the depletion of innovative skills and the difficulties of innovating under disequibrated conditions.
- Renewed equilibrium: Sooner or later the innovative impulse is insufficient to uphold the upswing. The downswing sharpens the competitive process of creative destruction, where many old firms are selected out of the economic system. At the end a renewed and well-established routine system emerges.
- Evolution: The economic evolution of the routine system consists in a series of routinised equilibria and innovative disturbances.

We may clarify this highly simplified process of creative destruction in terms of the innovation effect in one period and the selection effect in the subsequent period.<sup>13</sup> But let us first try to describe cyclical creative destruction as a pure selection process at the level of subpopulations of adopters and non-adopters. According to this interpretation (which has some support in Schumpeter's *Business Cycles*) the initial state of the economic system has no significant variance with respect to productivity. However, the population consists of two subpopulations: a very small subpopulation that has already adopted a productivity-enhancing innovation and a dominant subpopulation that has not. During the upswing the subpopulation of adopters increases its weight, both by expansion of the existing firms and by recruitment of other firms that apply the innovation. Thus there is a selection effect during the upswing. Here the adopting subpopulation is facing positive selection in both relative and absolute terms, and the non-adopting subpopulation is at least facing negative relative selection but probably also negative absolute selection.

But firms and their employees are not interested in what happens at the level of subpopulations. Their main problem is whether they individually can avoid negative selection (cf. equations (2) and (3)). During the upswing this negative selection can be avoided in two ways. First, non-adopters may avoid absolute negative selection because of an inflow of unutilised resources to the economic system so that the mean reproduction coefficient  $w > 1$  (Schumpeter normally assumes away this possibility). Second, they may avoid the absolute decline of their resources by becoming adopters (thereby contributing to the firm-level innovation effect). During the downswing these

possibilities becomes harder to exploit. First, the mean reproduction coefficient tends to be less than unity, and at the same time product markets and financial markets become more selective. Second, the firms that have not already become adopters during the upswing are the ones that are likely to have large difficulties in adopting. Therefore, the downswing a period where absolute creative *destruction* becomes widespread. Since economic agents normally react more strongly to negative selection than to positive selection, this destruction immediately points toward Schumpeter's (1950) vision of long-term socio-political consequences of the process of creative destruction. In evolutionary terms the downswing is the period in which the productivity-enhancing innovation spreads throughout the system so that behavioural variance reaches a minimum. But Schumpeter allows for a limited presence of the innovation that will carry the next wave of evolutionary change. The problem is, of course, why this innovation takes place in firms (or plants) that only have few resources and thus do not contribute substantially to the variance. But this question brings us beyond the limits of the present paper.

#### *4.3. Revisiting Schumpeter's long-term evolutionary trends*

Although Schumpeter's analytical scheme clearly expresses his basic vision of the process of creative destruction and although he thought much about the long-term change of this process, he never made his ideas empirically operational. First, the scheme assumes a sequence of equilibria that is seldom found in reality. Here innovation also takes place in relatively disequibrated environments, and thus the system never becomes fully routinised. Instead innovation and selection take place in parallel, but the cyclical movement of the effects may still be present in economic life (see equation (9)). Second, old firms are not always mere adaptors or helpless victims in the process of creative destruction. On the contrary, they might renew themselves to a degree that makes them winners in the process of selection. In the present context the interesting problem is whether the subpopulation of negatively selected firms is able to produce supernormal innovative-response-to-decline effects (see equation (3)). Third, firms tend to reorganise themselves in ways that make them adapted to operate in an environment characterised by permanent evolution. In such an environment innovation becomes a more or less permanent activity, and thereby firms may avoid creative destruction—at least in the short and medium term. But Schumpeter's formulations represent hardly more than a scenario of a fully routinised process of

innovation. The real question is whether and to which extent the aggregate selection effect decreases over time and even becomes negligible compared to the innovation effect. Schumpeter (1939; 1950) recognised these and other limitations to the formality and testability of his analytical scheme and his hypotheses, but he was never able to provide an alternative that allowed him to approach the evometric account for creative destruction.

In terms of evometrics we may, for instance, try to clarify the much-discussed change from Mark I to Mark II of Schumpeter's analysis of creative destruction under capitalism. If all innovative activities are transformed from individual entrepreneurs that innovate by creating new firms to oligopolistic firms with permanent in-house innovation, then we should expect to see that an increasing part of evolutionary change is due to the innovation effect while a decreasing part is due to the selection effect. The reason is that such oligopolistic firms do not wait with their reactions until they are selected away. Instead they use innovation as a means of keeping up with the mean behaviour of the population of firms. Thus what in an earlier phase of capitalism was obtained through the selection effect will now be obtained through the innovation effect. Since this proposition is not generally obvious, we seriously need empirical studies about the issue. In these studies we will have much need of the multi-level version of evolutionary change. The reason is that the Schumpeterian large-scale firms consist of many units, and some of the apparent disappearance of the selection effect may be due to a movement from selection between firms to selection within firms. It is, however, on balance likely that we shall find an increased importance of the innovation effect as a partial substitute for the selection effect.

Nelson and Winter's (1982, chs 12–14) simulation models of Schumpeterian competition may be seen as extreme versions of the Mark II account for creative destruction without destruction. With respect to these models Nelson and Winter only apply statistical analysis to find the typical behaviour of simulation runs instead of studying particular runs that are heavily influenced by random events. The application of evometrics implies a deeper analysis of the simulated evolutionary process. It immediately reveals that in the long run the Nelson–Winter models are dominated by the innovation effect. The reason is that the large firms show monopolistic restraint with respect to investment. Therefore, they do not transform super-normal productivity into super-normal reproduction coefficients. Instead they draw profits out of the industry. In such a setting there emerges a sustainable variance with respect to

productivities, since the covariance between reproduction coefficients and productivities is small and since the population-level reproduction function is non-linear. Instead mean productivity change becomes dominated by the innovation effect.

## 5. Summary and conclusions

It is well known that Schumpeter had a very ambiguous relationship to econometrics. Although he played an important role in the establishment of the Econometric Society and tried to apply its research programme in his ambitious attempt to analyse the process of creative destruction in his *Business Cycles*, he nevertheless developed what may appear as a hostile evaluation of the work of the econometricians. In this paper it was argued that this ambiguity is due to his misplaced emphasis on aggregate time series. What Schumpeter needed was statistical tools for performing the analysis of the aggregate effects of evolution in terms of the underlying population dynamics. These tools have been developed within biometrics, and they have recently become directly applicable to economic evolution due to the development of a general evometrics. Central to evometrics is a method for partitioning evolutionary change developed by George Price. This method serves surprisingly well as a means of accounting for evolution and as a starting point for the explanation of evolution. The paper reviewed some of the basic elements of this evometrics and demonstrated how they can be applied to make Schumpeter's theories operational. Although Price (1970, p. 521) remarked that '[r]ecognition of the covariance is of no advantage for numerical calculation, but of much advantage for evolutionary reasoning and mathematical model building', one of the main advantages is that his approach gives us a means of formulating theories and models in a quantitative way that, at least in principle, corresponds to measurable aspects of real evolutionary processes. This potential movement from theory to empirical studies was illustrated by the discussions of the theoretical problems of Schumpeter in handling creative destruction in a quantitative way and by the relative ease in specifying some of his theories and the related empirical questions in evometric terms.

Ultimately, the basic argument for evometrics is that the partitioning of the process of economic evolution increases our ability to handle empirical questions. Here four areas of study were touched upon. First, we need to confront the question of the relative strengths of the selection effect and the innovation effect in the process of

creative destruction. In contrast to the situation in evolutionary biology, the innovation effect is likely to be large and it may in principle explain total productivity change. Second, we need to decompose any given innovation effect as much as possible to deepen our understanding of creative destruction. Thus regional innovation effects should, if possible, be split so that we see how much selection takes place between firms. Firm-level innovation effects should be split so that we consider how much selection takes place between plants with different productivities. Even plants-level innovation effects may to some extent be partitioned. Since the economy is not organised in neatly hierarchical levels, this exercise is not easy; but we are allowed to try out different partitionings. Third, we should split the different effects into components that relate to firms that grow or decrease (relatively or absolutely). Thereby we may study the different productivity responses of the different subpopulations. In this study the emphasis on productivity issues gives a good deal of concreteness. For instance, Simon's conjecture that an unsatisfactory performance lead to a super-normal improvement of productivity needs to be specified, and common sense suggests that this and other 'Lamarckian' conjectures cannot hold in general. Fourth, we should study concretely under which conditions productivity leadership does and does not lead to increasing employment shares. Thus we should try to partition the selection efficiency into different sub-efficiencies. For instance, we need to study statistically the transformation of price or quality differentials to market share change in the product market, the transformation of costs into prices (where productivity leaders may have higher mark-ups), and the transformation of labour productivities to costs (where wages may be higher among productivity leaders). Future will show whether the systematic analysis of the data in terms of these four and several further empirical questions will significantly enhance our understanding of the actual processes of creative destruction.

## Acknowledgements

This paper is partly based on a working paper that was presented at the Third Workshop on the Economic Transformation of Europe (ETE), Sophia-Antipolis, 29–30 January 2004. This ETE paper had the title 'Evolutionary econometrics: From Joseph Schumpeter's failed econometrics to George Price's general evometrics and beyond'. Important comments have been given by Thomas Brenner, Uwe Cantner, Stan Metcalfe and Ulrich Witt.

## Notes

<sup>1</sup> As demonstrated below, this toolbox owes much to George Price and his equation for the partitioning of evolutionary change. Metcalfe (2002, p. 90) has remarked that '[f]or some years now evolutionary economists have been using the Price equation without realising it' and similar statements are made in a more developed form by Knudsen (forthcoming). Such statements hold for Metcalfe's (1998; 2001) contributions to a statistically oriented evolutionary economics and for the discussion of group selection within evolutionary game theory (Bowles, 2004, ch. 13), but they also have some truth for Nelson and Winter's (1982) pioneering contribution to the field. Even in applied economics with no evolutionary pretensions, we find a groping toward a general evometrics.

<sup>2</sup> Cantner and Krüger (2004) have suggested that the toolbox should be called 'evolometrics'. This suggestion is based on their attempt to systematise different contributions to applied evolutionary economics. For the moment I shall, however, stick to the name 'evometrics'. The main reason is that this name emphasises the relationship to biometrics, which is still the dominant contributor to the development of measurements and analyses of evolution. But it should be emphasised that a name is just a convention, and in the long run the evolving naming convention will determine whether the toolbox will be called 'evolometrics', 'evometrics', or something else.

<sup>3</sup> The conflict between on the one hand Tinbergen and on the other hand Schumpeter and Keynes is described by Louca (1997, 260–267).

<sup>4</sup> Bresnahan (1992) has inspired this analogy.

<sup>5</sup> Although Price did not use the terms of evolutionary metrics and evometrics, they seem clearly to describe his research programme. This research programme was described in a grant proposal for the Science Research Council of Great Britain in 1969 (Frank, 1995, 384–386) and, to some extent, in a posthumously published paper by Price (1995). I have chosen the term 'evometrics' partly to emphasise its relationship to the biometric tradition of Galton, Pearson and Fisher. Price worked both intellectually and physically in that tradition (for instance, he had a working place in the Galton Laboratories in London), but he was at the same time able to transcend the tradition to overcome its heavy connection to genetics. But he had some previous experience in e.g. game theory and social science, and he developed his ideas in relation to the path-breaking work on the biological evolution of social behaviour by Hamilton (1996).

<sup>6</sup> It should be noted that Price's approach is only one of several approaches to the formal analysis of evolution. Other approaches are Eigen's quasi-species equation, the replicator equation, the replicator-mutator equation and the adaptive dynamics approach. However, Page and Nowak (2002) prove that the approaches are compatible and that Price's equation plays an important role in unifying the formalisms of evolutionary dynamics.

<sup>7</sup> In the present notation means are denoted without the usual bars, so that  $w = \bar{w}$ . This is important when we later perform multi-level analysis. What at the level of firms appear to be a single number like  $w_i$  will often show up to be a mean of plant-level values, i.e.  $w_i = \sum_j s_{ij} w_{ij}$ .

<sup>8</sup> Fisher's (1999) formal analysis of evolution did not include this effect, and thereby much confusion arose about his fundamental theorem of natural selection (see Price, 1972b; Frank, 1997). However, Fisher (1999, 41–42) treated the effect verbally and called it the 'deterioration of the environment'. Hereby he emphasised that the increase in mean fitness due to the promotion of individuals with the 'best' characteristics cannot in the long run imply an expansion of the whole population (the species or the industry). Thus the individual reproduction coefficients cannot be the same in the second period as they were in the first period.

<sup>9</sup> Conner and Hartl (2004, ch. 6) give basic description of the issues and formalisms involved in the analysis of quantitative fitness functions.

<sup>10</sup> This definition of evolution is not complete. It ignores the cases of stabilising and disruptive selection in which the variance is changed while the mean does not change.

<sup>11</sup> It was Hamilton (1996, 332–337) who in 1975 made the first systematic application of Price's equation for studying the combined effects of group-level selection and individual-level selection. Later applications are covered by Sober and Wilson (1998) and Henrich (2004). A deeper question is that is treated by means of Price's expanded equation is how individual selection at one level promotes the evolutionary transition to organisational solutions at the next higher level (Michod, 1999).

<sup>12</sup> Actually, they refer not to productivity but to the natural logarithm of total factor productivity.

<sup>13</sup> The endogenous growth models of Aghion and Howitt (1998) represent an extreme stylisation of this process: each innovation is the basis for a monopoly that is destroyed by the next innovation.

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## Appendix: Formal partitioning of evolutionary change

### *The basic version of Price's equation*

The derivation of the different versions of Price's equation that are used in the present paper is fairly simple. We simply exploit the definitions given in tables 1 and 2. Here are the first steps:

$$\begin{aligned}
 \Delta z &= z' - z = \sum s'_i z'_i - \sum s_i z_i \\
 &= \sum (s_i + \Delta s_i)(z_i + \Delta z_i) - \sum s_i z_i \\
 &= \sum \Delta s_i z_i + \sum (s_i + \Delta s_i) \Delta z_i.
 \end{aligned} \tag{A1}$$

The task is to rewrite the two terms of equation (A1) into the components of Price's equation. Before we do so, we note that

$$\sum \Delta s_i w = 0 \tag{A2}$$

and

$$s'_i = x'_i / x' = x_i w_i / x w = s_i w_i / w. \tag{A3}$$

We are now ready to rewrite the first term of equation (A1):

$$\begin{aligned}
 \sum \Delta s_i z_i &= \sum \Delta s_i z_i - \sum \Delta s_i z \\
 &= \sum \Delta s_i (z_i - z) \\
 &= \sum (s'_i - s_i)(z_i - z) \\
 &= \sum s_i (w_i / w - 1)(z_i - z) \\
 &= 1/w \sum s_i (w_i - w)(z_i - z) \\
 &= \frac{\text{Cov}(w_i, z_i)}{w}.
 \end{aligned} \tag{A4}$$

In this rewrite we used equation (A2) in line 1 and equation (A3) in line 4.

Similarly we rewrite of the second term of equation (A1):

$$\begin{aligned}
\sum (s_i + \Delta s_i) \Delta z_i &= \sum s'_i \Delta z_i \\
&= \sum s_i (w_i / w) \Delta z_i \\
&= 1/w \sum s_i w_i \Delta z_i \\
&= \frac{E(w_i \Delta z_i)}{w}.
\end{aligned} \tag{A5}$$

In this rewrite we used equation (A3) in line 2.

By inserting the results of equations (A4) and (A5) in equation (A1), we obtain Price's equation:

$$\Delta z = \frac{\text{Cov}(w_i, z_i)}{w} + \frac{E(w_i \Delta z_i)}{w}. \tag{A6}$$

#### *Fisher's theorem as a special case of Price's equation*

Fisher's so-called fundamental theorem of natural selection is a special case of Price's equation (A6). In this equation  $z_i$  can be defined as any characteristic. If we consider the reproduction coefficient  $w_i$  as the characteristic under study, equation (A6) becomes

$$\begin{aligned}
\Delta w &= \frac{\text{Cov}(w_i, w_i)}{w} + \frac{E(w_i \Delta w_i)}{w} \\
&= \frac{\text{Var}(w_i)}{w} + \frac{E(w_i \Delta w_i)}{w}.
\end{aligned} \tag{A7}$$

Under the assumptions that  $w=1$  and  $\Delta w_i = 0$  for all  $i$ , equation (A7) simplifies to

$$\Delta w = \text{Var}(w_i). \tag{A8}$$

#### *Multi-level expansion of Price's equation*

We multiply equation (A6) by  $w$  to obtain

$$w \Delta z = \text{Cov}(w_i, z_i) + E(w_i \Delta z_i). \tag{A9}$$

If  $w_i = \sum_j s_{ij} w_{ij}$  and  $z_i = \sum_j s_{ij} z_{ij}$ ,  $w_i \Delta z_i$  can be partitioned by equation (A9):

$$w_i \Delta z_i = \text{Cov}(w_{ij}, z_{ij}) + E(w_{ij} \Delta z_{ij}). \tag{A10}$$

By inserting equation (A10) into equation (A9), we see that

$$\begin{aligned}
w \Delta z &= \text{Cov}(w_i, z_i) + E \left[ \text{Cov}(w_{ij}, z_{ij}) + E(w_{ij} \Delta z_{ij}) \right] \\
&= \text{Cov}(w_i, z_i) + E \left[ \text{Cov}(w_{ij}, z_{ij}) \right] + E \left[ E(w_{ij} \Delta z_{ij}) \right].
\end{aligned} \tag{A11}$$

If  $w_{ij}$  and  $z_{ij}$  are mean values, the last term of equation (A11) can be partitioned by means of equation (A9).

#### *Further partitioning of the versions of Price's equation*

In some cases it is relevant to split populations into subpopulations of increasing and decreasing entities:  $i \in C$  if  $w_i \geq 1$ ;  $i \in D$  if  $w_i < 1$ . Given this definition of subpopulations and by defining sums with no elements as zero, equation (A9) can be further partitioned:

$$\begin{aligned}
w \Delta z &= \text{Cov}(w_i, z_i) + E(w_i \Delta z_i) \\
&= \sum s_i (w_i - w)(z_i - z) + \sum s_i w_i \Delta z_i \\
&= \sum_{i \in C} s_i (w_i - w)(z_i - z) \\
&\quad + \sum_{i \in D} s_i (w_i - w)(z_i - z) + \sum_{i \in C} s_i w_i \Delta z_i + \sum_{i \in D} s_i w_i \Delta z_i.
\end{aligned} \tag{A12}$$

If we divide equation (A12) by  $w$ , we may name the four terms into which the total change ( $T$ ) is partitioned:

$$\Delta z = T = S_C + S_D + I_C + I_D. \quad (\text{A13})$$

The terms of equation (A13) do not reflect simple statistical concepts. But in the social sciences it is sometimes relevant to study  $S_C + I_C$  and  $S_D + I_D$ .

We may perform similar partitioning and analysis of equations (A7) and (A11).