Starting the Study of Economic Evolution with Nelson–Winter-like Models

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Overview

Purposes of the lecture and the slides

- **Introduction to Nelson–Winter models** which has a pioneering status in evolutionary modelling
- **Introduction to evolutionary modelling** by constructing from scratch a family of Nelson–Winter-like models
- **Introduction to evolutionary simulation** in relation to Nelson–Winter-like models
- **Compendium** of Andersen and Valente (2002), Andersen (2001) and—to some extent—Valente and Andersen (2002). References are found on the last slide

Contents of the slides and—to some extent—the lecture

- **The Nelson–Winter book:** background, main models and problems
- **The standard Nelson–Winter model** of industrial dynamics (member of the AK model family)
- **The Nelson–Winter-like AL models** with only labour and knowledge (the AL model family)
  - **The AL Mark I model** with fixed productivities and replicator dynamics—including deductions and simulations
  - **The AL Mark II models** with endogenous change of productivities
  - **The AL Mark III models** with emerging markets for intermediate goods and R&D specialisation
- **The art of simulation** in relation to evolutionary models and the Laboratory for simulation development (Lsd)

The history and state of evolutionary economics

Stylised history in three stages (cf. Hodgson 1993 and Andersen 1996):

1. **Old evolutionary economics.** The verbal accounts that can now be recognised as covering aspects of economic evolution
   - From Adam Smith via Marx, Menger and Marshall to Veblen, Schumpeter and Hayek
2. **"Dark Ages" of evolutionary economics.** The crowding out of the verbal approach when economics reached high standards in static analysis
   - Especially from the 1920s. Lionel Robbins: the old stuff is "intelligent after-dinner talk"
3. **New evolutionary economics.** The modern studies that are based on a break-through in evolutionary modelling

   a. Starting points not least in three books:
   - Nelson and Winter, *An Evolutionary Theory of Economic Change* (1982)
   - Maynard Smith, *Evolution and Game Theory* (1982)
   - Axelrod, *The Evolution of Cooperation* (1984)

   b. Growth and diversity of new evolutionary economics
   - Neo-Schumpeterian studies of technology-based industrial dynamics and economic growth
   - Evolutionary game theory of the formal type that study the evolutionary selection of Nash equilibria
   - Simulation-oriented studies of the evolution of institutions and dominating strategies
   - Complexity studies that emphasises of the path dependency of dominant technologies, etc.
   - And so on ...

   c. Contemporary problems and perspectives
   - The production of solid results of evolutionary analysis is still rather slow
   - Relatively high barriers to entry to evolutionary modelling and simulation
   - Rapid diffusion (e.g. into the journals of business economics and economic geography) before basic issues have been clarified
   - The diffusion into policy aspects is increasing, and here the modelling problems are even larger

Revisiting the Nelson–Winter book 20 years after

Contents in brief summary

Part I	Overview and Motivation
Part II	Organization-Theoretic Foundations of Economic Evolutionary Theory
Part III	Textbook Economics Revisited
Part IV	Growth Theory
Part V	Schumpeterian Competition
Part VI	Economic Welfare and Policy
Part VII	Conclusion

Characteristics (cf. Andersen 1996, Ch. 4)

The "population perspective" is fundamental

- The NW evolutionary theory starts from recognising that firms are heterogeneous and boundedly rational, so they form a population with evolving characteristics

Synthesis of mechanisms. The NW evolutionary theory is based on a synthesis of

Transmission: Theories of routines and their reproduction in firms
Mutation: Theories of the creation of new routines in firms
Selection: Theories of the selection environment and its influence

Emphasis on simulation models. The synthesis of the evolutionary mechanisms is made by means of simulation models

- The starting point is Part II, but the actual synthesis takes place by means of the evolutionary NW models that are designed for computer simulation (Parts IV and V)

"Appreciative" approach. All major NW models are designed with a concrete problem in mind, although it is especially emphasised in recent "history-friendly" models

Analytical models as servants. Nelson and Winter make models for which mathematical deductions can be made *for the sake* of preparing the ground for more realistic models

Overview of Nelson–Winter models (numbered by chapters)

Simple formal models for analytical results:

NW6: Section "A Particular Model of Economic Selection": Nelson and Winter (1982, Ch. 6, 144–154)

NW7: Substitution "A Markov Model of Factor Substitution": Nelson and Winter (1982, Ch. 7, 175–192)

NW10.1: Development "Development and Backwardness in a Two-Technology Evolutionary Model": Nelson and Winter (1982, Ch. 10, 235–240, 240–245)

NW10.2: Growth as selection "Growth as a Pure Selection Process: Many Techniques and Many Variable Inputs": Nelson and Winter (1982, Ch. 10, 240–245).

Growth and industry models for simulation results:

NW9: Growth vs. Solow "An Evolutionary Model of Economic Growth": Nelson and Winter (1982, Ch. 9, 209–214)

NW12: Industrial dynamics I "Dynamic Competition and Technical Progress": Nelson and Winter (1982, Ch. 12, 281–287, 302 f.)

NW13: Industrial dynamics II "Forces Generating and Limiting Concentration under Schumpeterian Competition": Nelson and Winter (1982, Ch. 13)

NW14: Industrial dynamics III "The Schumpeterian Trade-off Revisited": Nelson and Winter (1982, Ch. 14)

Subsequent models by Nelson and Winter

XNW84: Regimes "Schumpeterian Competition in Alternative Technological Technological regimes": Winter (1984).

XNW99: History-friendly models " 'History-friendly' Models of Industry Evolution: The Computer Industry": Malerba, Nelson, Orsenigo and Winter (1999).

Limitations of the overview

Further works Neither analytical NW models published after 1982 nor the contributions of other authors are listed (the latter are partly surveyed in Kwasnicki, 2001).

The evolutionary mechanisms and the NW models (I)

Two core questions in NW models are:

Will the evolutionary process move toward monopoly?

How can this (unrealistic?) tendency be avoided?

The simple selection process due to the basic structure of NW models

- **Different productivities.** Firms normally have different capital productivities so that a given stock of capital can produce different quantities in different firms
- **Uniform prices.** All firms face the same output price and the same factor prices.
- **Different profit rates.** Different productivities and unitary prices means that firms have different profit rates
- **"Selection" of firms.** Firms are being "selected" since the different profit rates lead to different growth rates. Supernormal profits can be considered as rewards for high fitness, while subnormal profits are punishments for low fitness

Selection and the process of accumulation. Investment behaviour may change the basic selection mechanism

- **Simple capacity accumulation and monopoly.** If firms invest all their profits in accumulation and do not have external finance, then the firm with the highest productivity will grow into a monopoly
- **Monopolistic restraint.** In the standard NW model this full "selection" is avoided by monopolistic restraint on capacity accumulation. Thus the models include the firm's increasing awareness of the overall demand curve as its market share increases
- **Entry of new firms.** An alternative strategy to control the tendency toward monopoly is to introduce new firms into the industry. These new firms enter from an exogenous pool of firms (cf. e.g. Winter, 1984, 283–288).
- **External finance** could also change the basic selection process by sucking profits out of dominant firms. In the NW models it, however, strengthens the basic selection

The evolutionary mechanisms and the NW models (II)

The innovation–imitation process both produces and controls the variance of the firms' productivities

Innovation independent of firm behaviour. In this case innovations come like "mannah from heaven" to small and large firms. In this (unrealistic) way we can avoid monopolyt

Satisficing behaviour and innovation. In the NW9 growth model "search" for improvements is an ad hoc activity that emerges costlessly when a firm experiences unsatisfactory results. This behaviour helps to avoid monopoly

Improvement as a permanent strategy. This is the assumption in most of the NW models. The innovation–imitation process does not remove the long-term tendency toward monopoly because of increasing returns to the application of R&D results. If firms show no restraint on their expansion, monopoly emerges with a probability of one

"Schumpeterian" evolution includes jumps in productivity and quality, so here we see further possibilities to break out of near-monopoly situations

The standard NW model of industrial dynamics (I)

1. The markets are simplified to allow a focus on evolution

Capital and labour markets are are simply characterised by fixed prices and unlimited supply

The output market has a fixed demand D. The output price P_t is set so that the market clears, i.e.

$$P_t = D/Q_t \tag{1}$$

2. Production and profit for firm j in period t

Capital and productivity From period $t-1$ the firm has its own levels of capital $K_{j,t-1}$ and productivity $A_{j,t-1}$

Output Additional inputs (labour etc.) is bought, and the resulting output is

$$Q_{jt} = A_{j,t-1}K_{j,t-1} \tag{2}$$

Profit The sum of the output of all the firms $Q_t = \sum_{j=1}^{n} Q_{jt}$ results in the price P_t and the revenue P_tQ_{jt}. The fixed costs per unit of capital is c includes both wages and capital rental. There are also R&D expenditures that per unit of capital is r_j. Thus the profit is

$$\pi_{jt} = P_tQ_{jt} - (c + r_j)K_{j,t-1} \tag{3}$$

3. Capital accumulation is in principle simple

Depreciation Unless the firm invests its capital shrinks with the rate of depreciation δ.

Financial constraint The maximum investment is determined by the profit plus extra financial resources from banks. Thus $I_{jt}^{\max} = (1 + b)\pi_{jt}$.

Desired investment is determined by the size of price relative to the unit costs. If the expected mark up is less than unity, I_{jt}^{desire} becomes negative. If the firm has a large market share, it shows investment restraint because the way the output market is functioning. Thus I_{jt}^{desire} is also determined by the market share.

Actual investment per unit of capital comes from a combination of the constraint and the desire:

$$I_{jt} = \min(I_{jt}^{\max}, I_{jt}^{\text{desire}}) \tag{4}$$

The new level of capital is only for use in next period. It is:

$$K_{jt} = (1 + I_{jt} - \delta)K_{j,t-1} \tag{5}$$

The standard NW model of industrial dynamics (II)

4. **Endogenous productivity change** is the core issue of the NW model

   Productivity and R&D Capital productivity is determined by firm-specific knowledge that is improved by R&D that can be innovative and/or imitative. The R&D effort per unit of capital is split up according to a fix rule so that

$$r_j = r_j^{\mathrm{IN}} + r_j^{\mathrm{IM}} \tag{6}$$

   Imitative R&D gives—if successful—access to the best productivity of the industry $A_j^{\max}$. The total effort of the firm is $r_j^{\mathrm{IM}} K_{j,t-1}$ and the probabilistic productivity of research is λ^{IM}. The product of these two numbers gives the mean number of imitations per period. The actual number is drawn from a Poisson process. If the number is at least unity, we set $A_j^{\mathrm{IM}} = A_j^{\max}$; otherwise it is 0.

   Innovative R&D gives—if successful—access to a result draw from a normal distribution with a fixed standard deviation and a mean that differ between the different NW models; in the cumulative regime it is equal to the existing productivity. The total innovative effort of firm is $r_j^{\mathrm{IN}} K_{j,t-1}$ and the probabilistic productivity of research is λ^{IN}. The product of these two numbers gives the mean number of innovations per period. The actual number is the drawn from a Poisson process. If the number is at least unity, the firm gets a draw from the normal distribution and the result is A_j^{IN}. If innovative R&D is unsuccessful, this number is 0.

   New productivity is determined precisely and is available in the next period:

$$A_{jt} = \max(A_{j,t-1}, A_{jt}^{\mathrm{IM}}, A_{jt}^{\mathrm{IN}}) \tag{7}$$

   Appropriability regimes are specified directly by the productivity of imitative research. If this productivity is low, then the results of the innovators are well protected.

   Technological regimes are specified by the way the mean of the normal distribution of innovative results are determined: 1. The regime with firm-based cumulation of technology has a mean equal to $A_{j,t-1}$. 2. The regime with industry-based cumulation of technology has a mean equal to A_{t-1}^{mean}. 3. The regime with science-based technology has a mean equal to an exogenously knowledge that is exponentially growing.

Structure of the standard NW model

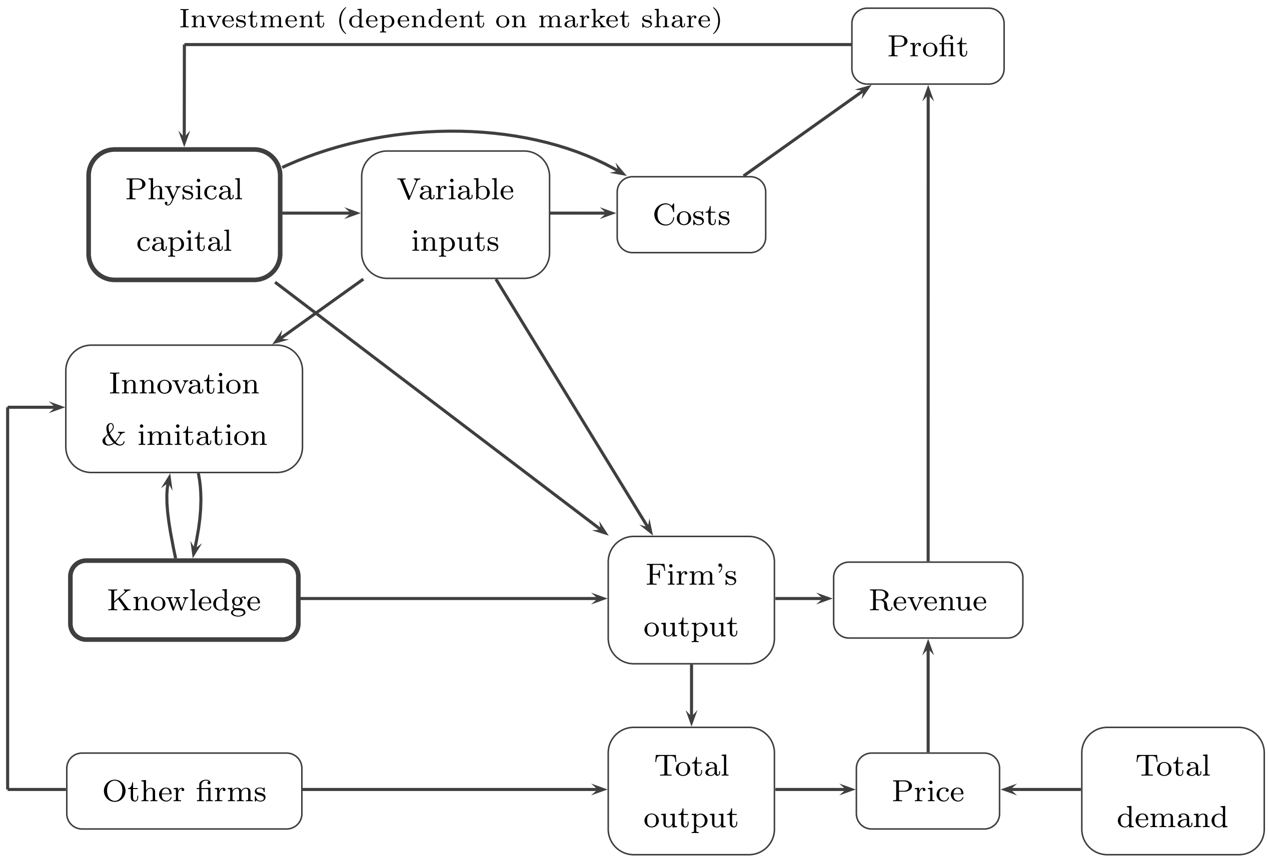


Figure 1: Structure diagram of the standard NW model of industrial dynamics

Strategies for modelling in the NW tradition

Results and problems The standard NW model has had a significant influence. But there are problems, e.g.

- **Too many parameters.** Especially the fixed demand in standard NW models is problematic
- **Ad hoc investment restraint** of large firms is problematic *as a starting point*
- **Ad hoc entry of new firms** in extended NW models like Winter (1984)
- **Separate simplified models** to study replicator dynamics, etc. are problematic
- **Difficulties of extended simulation models** because of the complexities
- **Difficulties of teaching** because of the complexities

Solutions. There are different ways of coping with the problems within the NW tradition, e.g.

1. **The back-to-basics strategy** seeks for highly simplified starting points and then moves very slowly to the real complexity of evolutionary processes.
2. **The extensions strategy** is largely related to Winter (1984). It has run into decreasing returns ...
3. **The "history-friendly" strategy** of Malerba, Nelson, Orsenigo and Winter (1999) emphasises the main NW strength: the relationship to ambitious empirical studies
4. **The demand side strategy** adds a theory of the complexity of demand to reach more realistic industrial dynamics (partly in relation to the Chamberlin–Lancaster traditions)
5. **The multiactivity generalisation** starts from complex firms with intraorganisational diversity to deal with intermediate exchange, specialisation of R&D, and multisectoral growth and development

In the following strategy 1 is chosen, but it is intended to support other strategies (especially strategy 5).

Constructing the AL model family in three steps

1. **Study the problems of the NW model family.** This model family may be called the AK models of economic evolution.[a] To sum up, major problems are
   - The de facto standard is the AK model of industrial dynamics
   - The AK growth model has been less developed
   - Specialised AK models analyse aspects of formal evolution independently from the rest of the models
   - These types of model should be integrated
   - But physical capital hinders an easy introduction of multiple activities and the introduction of fissions and fusions of firms
2. **Develop a pure-labour version of the NW model of growth.** This model family may be called the AL models
   - The AL models also cover industrial dynamics and formal replicator dynamics
   - AL models make it easy to introduce multiple activities as well as fissions and fusions
   - The AL models is handy tools for teaching evolutionary models and simple simulation techniques
3. **Develop multiactivity AL models** as a generalisation of simple and well-studied AL models
   - It inherits all the characteristics of the simple AL model, but is more complex
   - It allows us to start from a study of intraorganisational diversity
   - We can gradually introduce
     - intermediate exchange without adding ad hoc demand specifications
     - specialisation of R&D in a rich selection environment
     - multisectoral growth and development from the bottom up

[a] Because of the production function $Q_{jt} = A_{j,t-1}K_{j,t-1}$

The fixed-productivity AL model of growth (AL Mark I)

The households and the wage rate. There is an unchanging number of N households, which each supply one unit of labour. The wage rate is fixed to unity, $w = 1$. Households spend all income on the single product of the economy.

The firms. There is a fixed number of n firms. We consider firm j in period t.

Employees and productivity The firm has a stock of employees $L_{j,t-1}$ and fixed productivity A_j. All employees produce at the productivity level of the firm. The unit cost of the firm is $c_j = 1/A_j$.

Output is produced according to

$$Q_{jt} = A_j L_{j,t-1} \tag{8}$$

Profit The output of the firm is sold at the market price P with the revenue PQ_{jt} and the costs $wL_{j,t-1} = L_{j,t-1}$. Thus the profit is

$$\pi_{jt} = PQ_{jt} - L_{j,t-1} \tag{9}$$

New employment The payment of the firm's employees ($L_{j,t-1}$) is made out of the revenue from period $t-1$, so all the revenue of period t is used for employment. Thus the firm in period t will employ $L_{jt} = PQ_{jt}$. The change in the number of employees is

$$\Delta L_{jt} \equiv L_{jt} - L_{j,t-1} = PQ_{jt} - L_{j,t-1} = \pi_{jt} \tag{10}$$

The markets. There are only two markets: the output market and the labour market.

Output demand in period t is simply the income of the households. Thus the output market's demand $D_t = \sum_{j=1}^{n} L_{j,t-1} = L_t$.

Output supply is the aggregate output of the firms, $Q_t = \sum_{j=1}^{n} Q_{jt}$.

Output price The price is set to clear the market. To secure that the supply Q is bought, we have the price $P_t = D_t/Q_t$

Labour supply is simply the number of households N.

Labour demand is determined by the firms. The aggregate demand is $L_t = \sum_{j=1}^{n} P_t Q t j$

Labour market equilibrium. Since the model provides no mechanism to bring the fixed supply of labour in accordance with the demand for labour, the question is whether $N = L_t$. We shall return to this question.

Summary of the AL Mark I model

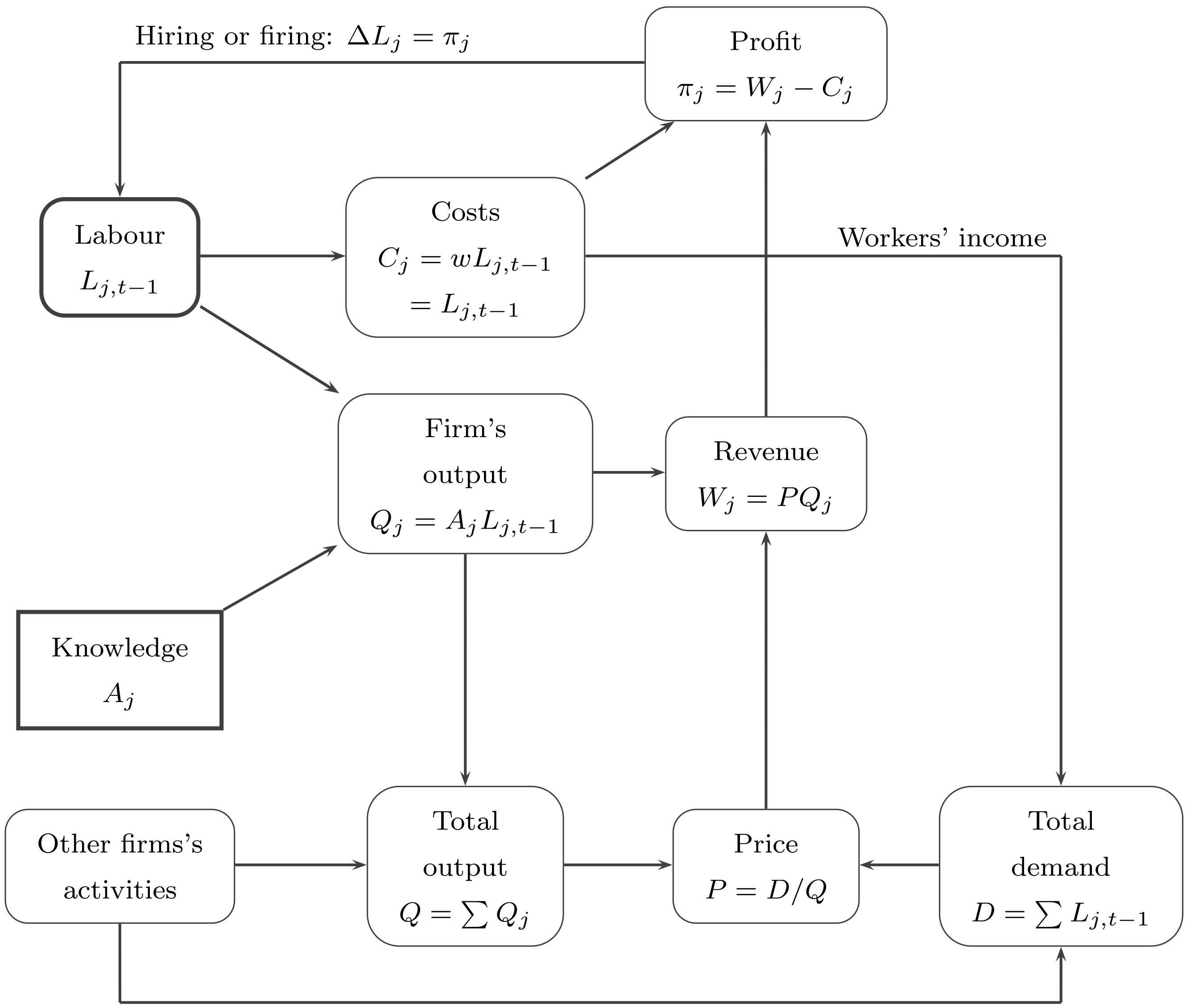


Figure 2: Structure diagram of the AL Mark I model. For simplicity unlagged time subscripts have been omitted.

Deductions about the dynamics of the *AL* Mark I model

Simplicity leads to deductions. Since the *AL* Mark I model is designed according to the KISS principle, we can make formal deductions about its dynamics:

1. **The theorem of constant employment.** Aggregate employment and aggregate demand is unchanging in the *AL* Mark I model.
2. **The theorem of distance-from-mean dynamics.** In the *AL* Mark I model the growth rate of the market share of any firm is equal to the distance between the average unit costs and its unit cost times its market share, i.e. $ds_{jt}/dt = s_{jt}(\bar{c}_t - c_{jt})$.
3. **Fisher's fundamental theorem of natural selection.** In the *AL* Mark I model the growth rate of average productivity is equal to the negative of the variance of the market-share weighted unit costs, i.e. $d\bar{c}_t/dt = -\mathrm{Var}(c_{jt})$.

Deductions help in further work. Given these theorems we immediately can say a lot about the *AL* Mark I model.

Replicator dynamics. The distance-from-mean dynamics is also called replicator dynamics. It means that if all firms have mean productivity, there is no change in market shares. Furthermore, the firm with the highest productivity will always have the highest growth rate, so it will end up as a monopoly.

The importance of a statistical approach. The third theorem is a variant of the theorem of R.A. Fisher (creator of much of modern statistics and evolutionary analysis). It emphasises that we can perform our analysis in terms of the weighted mean variance of the unit costs/productivities. As variance moves towards zero, the change of market shares disappears.

Simulating the dynamics of the AL Mark I model (I)

Table 1: Hand simulation of the AL Mark I model.

Period	0	1	2	...	14	15
Firm 1 with $A_1 = 1.2$						
Employment	100.000	120.000	140.260	...	273.074	277.441
Output		120.000	144.000	...	321.468	327.689
Market share		0.400	0.468	...	0.910	0.925
Profit		20.000	20.260	...	5.185	4.366
Firm 2 with $A_2 = 1.0$						
Employment	100.000	100.000	97.403	...	25.523	21.609
Output		100.000	100.000	...	30.046	25.523
Market share		0.333	0.325	...	0.085	0.072
Profit		0.000	-2.597	...	-4.523	-3.914
Firm 3 with $A_3 = 0.8$						
Employment	100.000	80.000	62.338	...	1.403	0.950
Output		80.000	64.000	...	1.652	1.122
Market share		0.267	0.213	...	0.006	0.004
Profit		-20.000	-17.662	...	-0.662	-0.453
Aggregate data						
Employment	300.000	300.000	300.000	...	300.000	300.000
Output		300.000	308.000	...	353.165	354.334
Price/av. unit costs		1.000	0.974	...	0.849	0.847
Aggregate profits		0.000	0.000	...	0.000	0.000
Growth aver. costs		-0.027	-0.025	...	-0.003	-0.002
Variance of unit costs		0.027	0.025	...	0.003	0.003
Output dispersion (H)		2.9220	2.706	...	1.162	1.134
Instability (I)		0.177	0.169	...	0.025	0.022

Simulating the dynamics of the *AL* Mark I model (II)

Using the computer. Even in the simple *AL* Mark I model we get help

- **Run multiple steps.** Make a huge number of calculations to follow the dynamics to its end
- **Check different variance of productivities.** Generate distributions of productivities with a known variance and test the consequences
- **Be prepared for extensions.** The computer makes it easy to endogenise productivity change
- **Make graphical output** for comparisons and reporting

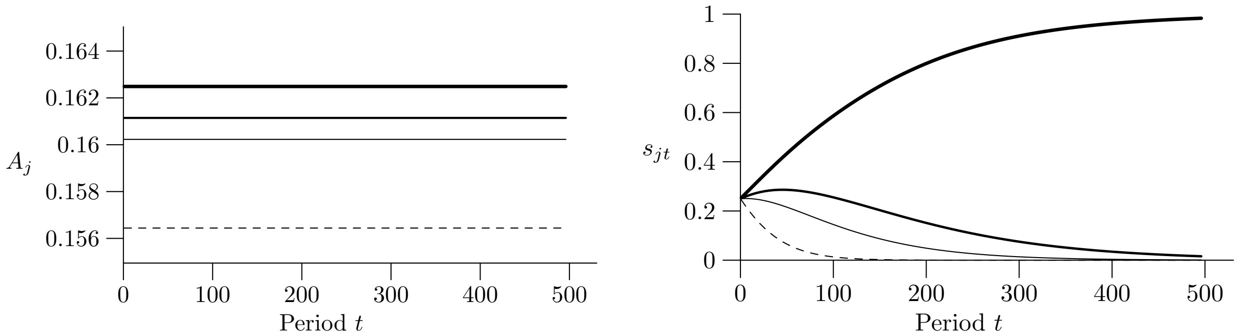


Figure 3: Simulation of the *AL* Mark Ia model for 500 periods with four firms. The productivities of the left panel have been drawn randomly from a distribution with a mean of 0.16.

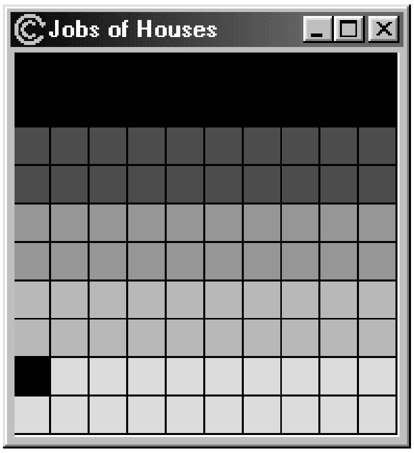
Economic life on the lattice

The lattice is a two-dimensional grid.

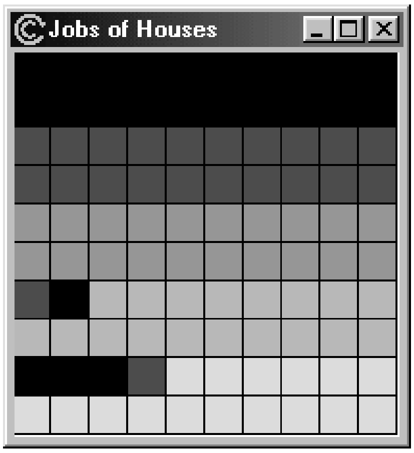
Households initially employed by the same firm are placed near to each other

Employment change is shown during the simulation

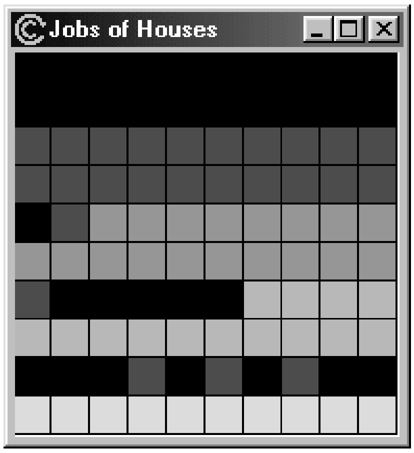
Productivity differences between employees may be introduced. The shift is always from low productivity to high productivity firms. But during a period new employees may gradually improve from the low level to the high level



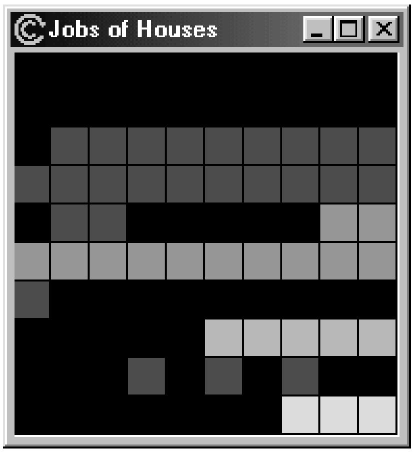
A. After 2 periods



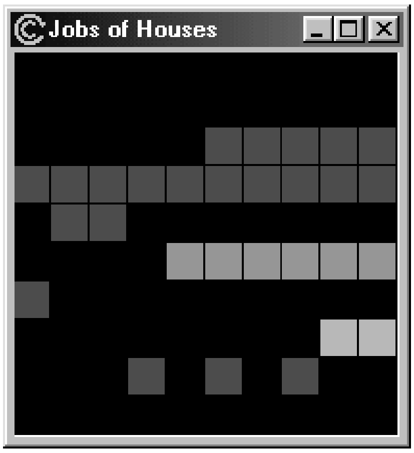
B. After 10 periods



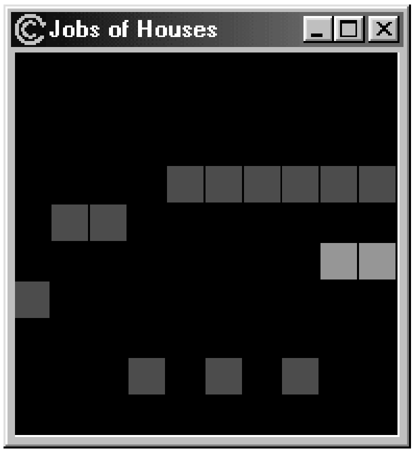
C. After 30 periods



D. After 80 periods



E. After 125 periods



F. After 200 periods

Figure 4: Evolution on the lattice of the *AL* Mark I model.

From the AL Mark I model to the AL Mark II models

Production and research is the new topic

Division of labour. The firm divides its stock of employees $L_{j,t-1}$ by a fixed parameter ρ_j. Labour for production is $L_j^{\text{prod}} = (1-\rho_j)L_{j,t-1}$ and labour for research is $L_j^{\text{res}} = \rho_j L_{j,t-1}$.

Production function. Only firm's production workers produce output

$$Q_j = A_j L_j^{\text{prod}} \tag{11}$$

Costs. Since $w = 1$, the firm's total costs are simply $C_j = L_j^{\text{prod}} + L_j^{\text{res}} = L_{j,t-1}$.

Profit. The firm sells all its output at the market price P. Thus it obtains the profit

$$\pi_j = PQ_j - L_{j,t-1} = PA_j L_j^{\text{prod}} - L_{j,t-1} \tag{12}$$

The R&D outcome is modelled as a two-stage stochastic process

Probability of a research success. The researchers have a fixed productivity that is measured as the average number of successes per period per researcher, $1/\lambda$. The result of the firm's total research activities is modelled as a Poisson process with average waiting time for a success equal to λ times the number of researchers.

Methods of research. The research workers apply different R&D methods according to fixed parameters that determine the degree to which the researchers focus on different ways of improving knowledge: (a) cumulation of the firm's own knowledge, (b) imitation of the leading firm in the industry, (c) application of the industry's average knowledge, and (d) application of general knowledge.

Outcomes of research. With cumulative knowledge the result is drawn from a normal distribution with mean determined by the firm's present productivity and a fixed standard deviation.

Productivity change. The firm's productivity in period t is the maximum of the existing productivity inherited from period $t-1$ and the potential productivity obtained by R&D, i.e.

$$A_j = \max(A_{j,t-1}, A_j^{\text{res}}). \tag{13}$$

Structure of the AL Mark II models with changing productivities

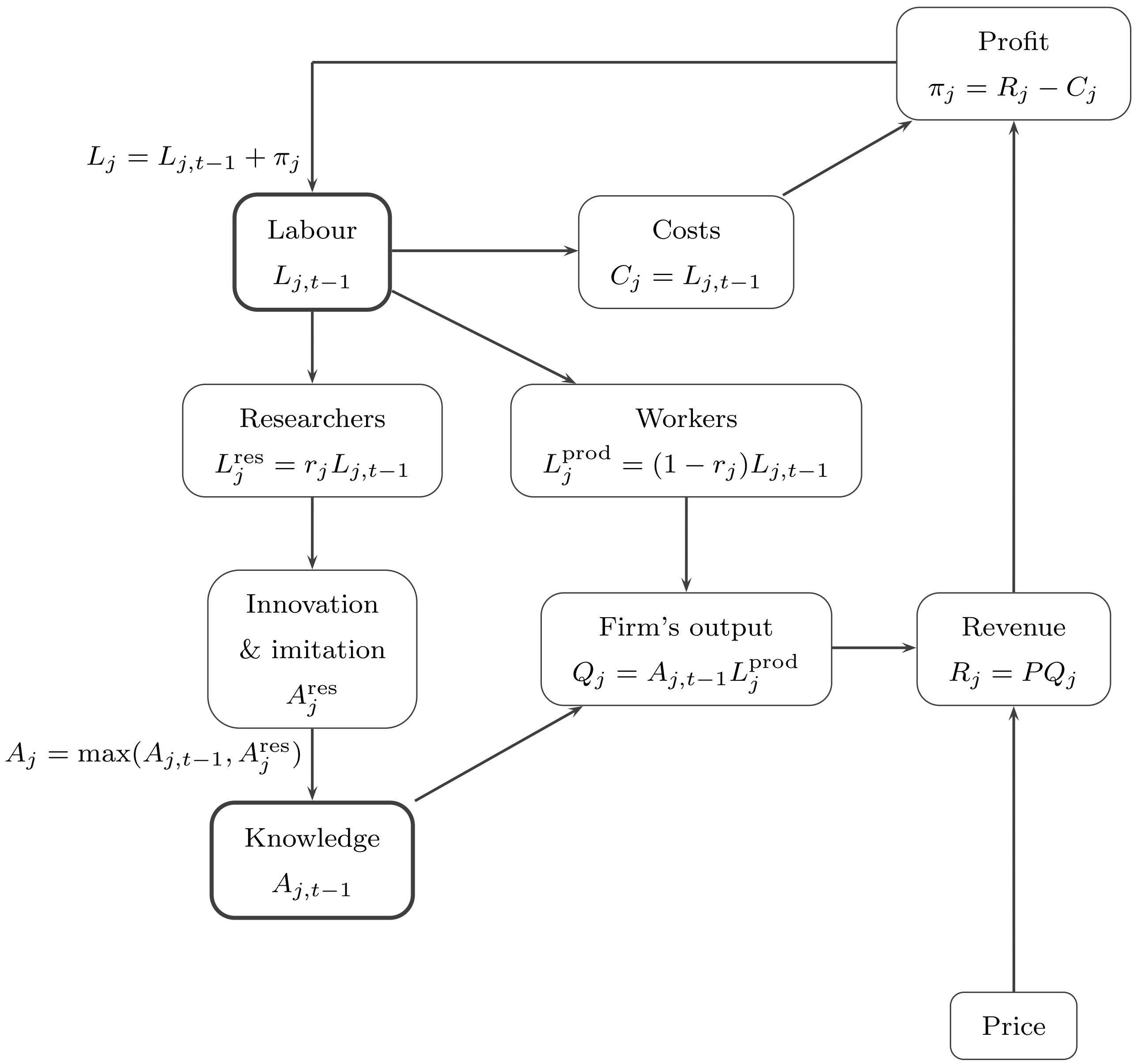
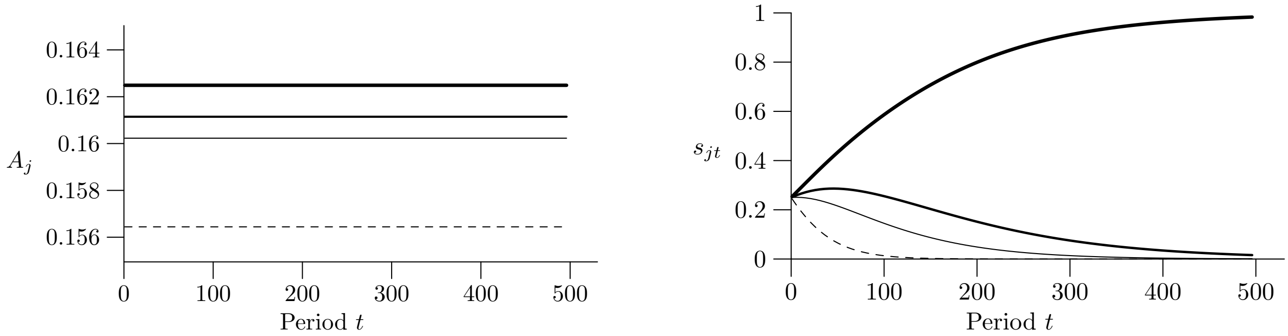


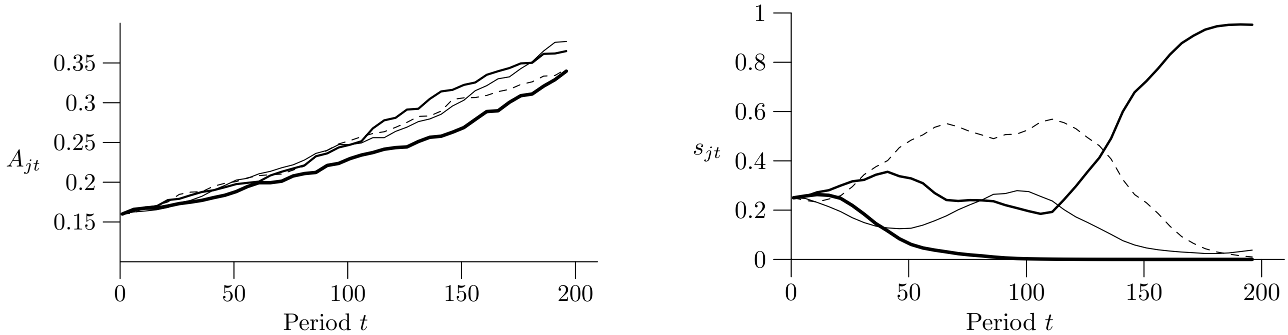
Figure 5: The basic structure of the AL Mark II family of growth models with endogenous determination of the productivity level (cf. figure 2). The diagram is drawn from the viewpoint of a particular firm j. The reading of the diagram starts from the state variables labour and knowledge (i.e. productivity). After the profit has been found, the employment level for the next period is determined. After innovation and imitation has been performed, the productivity of the next period is determined.

The dynamics of different *AL* Mark II models (I)

A. The case of fixed productivities (*AL* Mark Ia)



B. The case of random-walk cumulation of productivities (*AL* Mark IIa)



C. Longer run of the case of random-walk cumulation of productivities (*AL* Mark IIa)

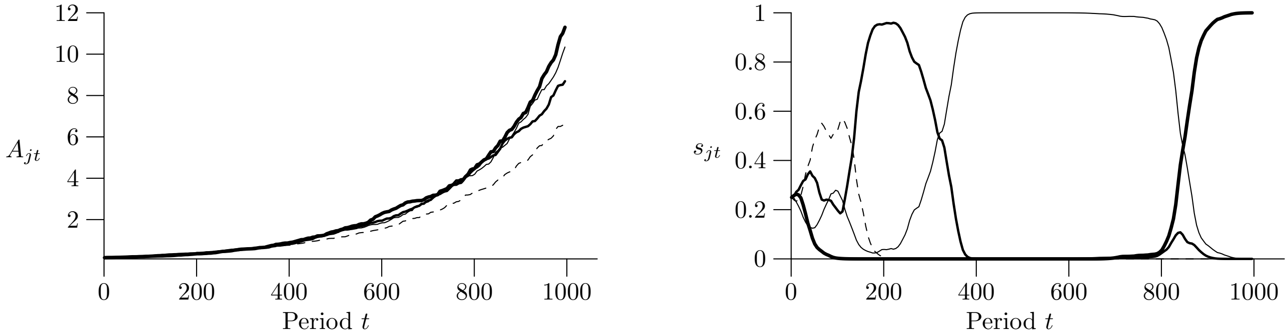
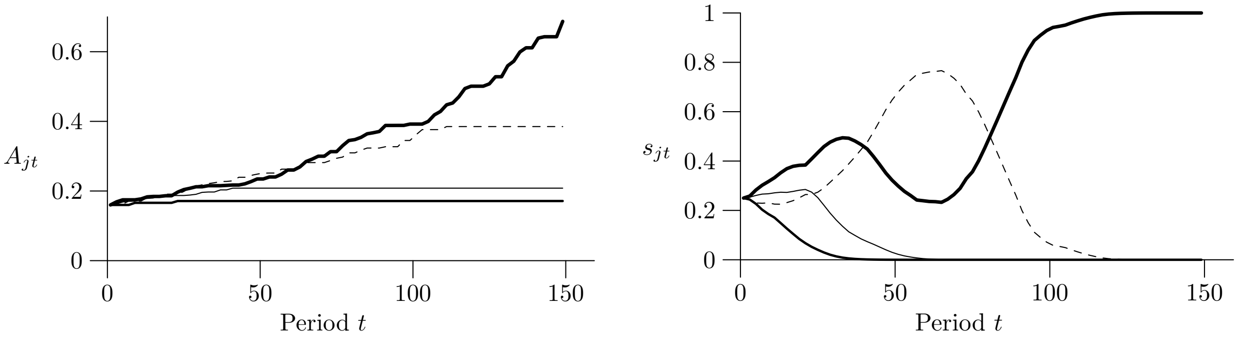


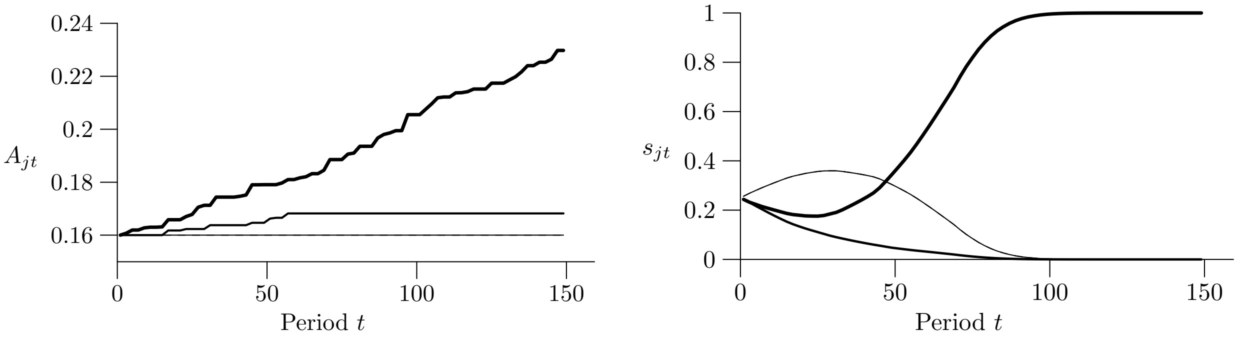
Figure 6: Simple dynamical patterns from simulations of *AL* Mark II models with four firms: (A) The research intensity to zero for all firms, so we have *AL* Mark Ia model. (B) In the *AL* Mark IIa model we have productivity increases that in each period is drawn from a random distribution with a mean equal to the present productivity level of the firm. (C) The specification of the *AL* Mark IIa model implies that a series of random successes can (with lesser and lesser probability) change the situation of panel B. Thus we in panel C consider a continuation of the previous *AL* Mark IIa simulation and see that panel B does not represent a lock-in situation.

The dynamics of different *AL* Mark II models (II)

D. The case of R&D-based cumulation of productivities (*AL* Mark IIb)



E. The case of two firms with R&D and two firms without R&D (*AL* Mark IIb)



F. The case of firms that are both innovators and imitators (*AL* Mark IIc)

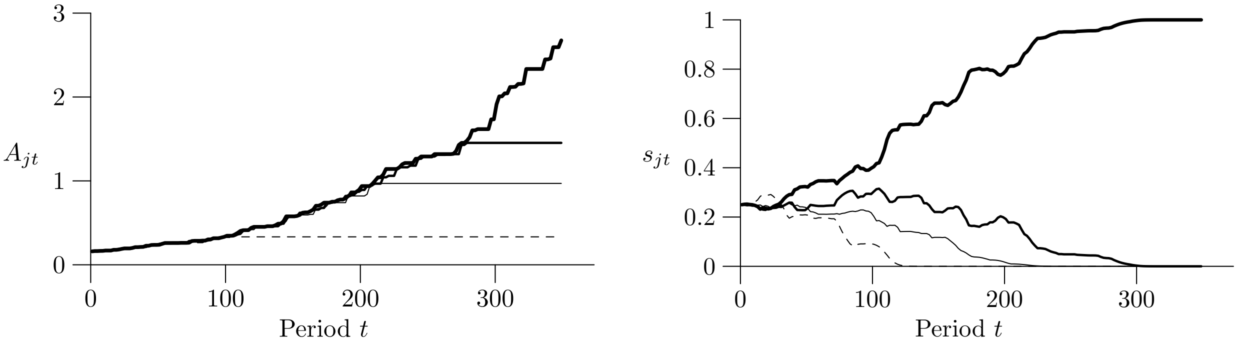


Figure 7: Dynamical patterns from simulations of *AL* Mark II models with four firms: (D) In the *AL* Mark IIb model firms may perform R&D, and the innovative results have their mean value in the present productivity of the firm. In panel D all firms perform have the same R&D intensity. (E) In panel E two firms perform R&D like in panels D, while the two other firms have no R&D and thus obtains an initially higher profitability. (F) In the *AL* Mark IIc model all firms may perform both innovation and imitation. Imitators can in one step reach the productivity level of the leading firm. In panel F all firms have the same intensity of innovative R&D and the same intensity of imitative R&D.

Towards *AL* Mark III with multiple products

- **Basic idea:** Move from the aggregate activities of *AL* Mark I and II firms to multiple activities
  1. Each firm j has m_{jt} production activities
  2. A similar number of R&D activities improve the knowledge on the production activities
  3. A structural R&D activity increases the number of production activities and the related R&D activities so that $m_{j,t+1} = m_{jt} + 1$
- **Simple interdependence of activities:**
  - One production activity produces final output like in the other *AL* models
  - $m_{jt} - 1$ production activities produce intermediate goods used in the final production activity
  - More complex interdependencies can easily be introduced
- **So what?**
  - We may start with single-activity firms and let multiple activities evolve
  - Two firms with equal aggregate productivities may differ in activity-level productivities
  - Firms can specialise into sellers and buyers of intermediate goods
  - Transaction costs
  - R&D strategies may also be specialised
  - Different R&D strategies fit different exchange regimes

The firm in the *AL* Mark III models of growth and industrial dynamics

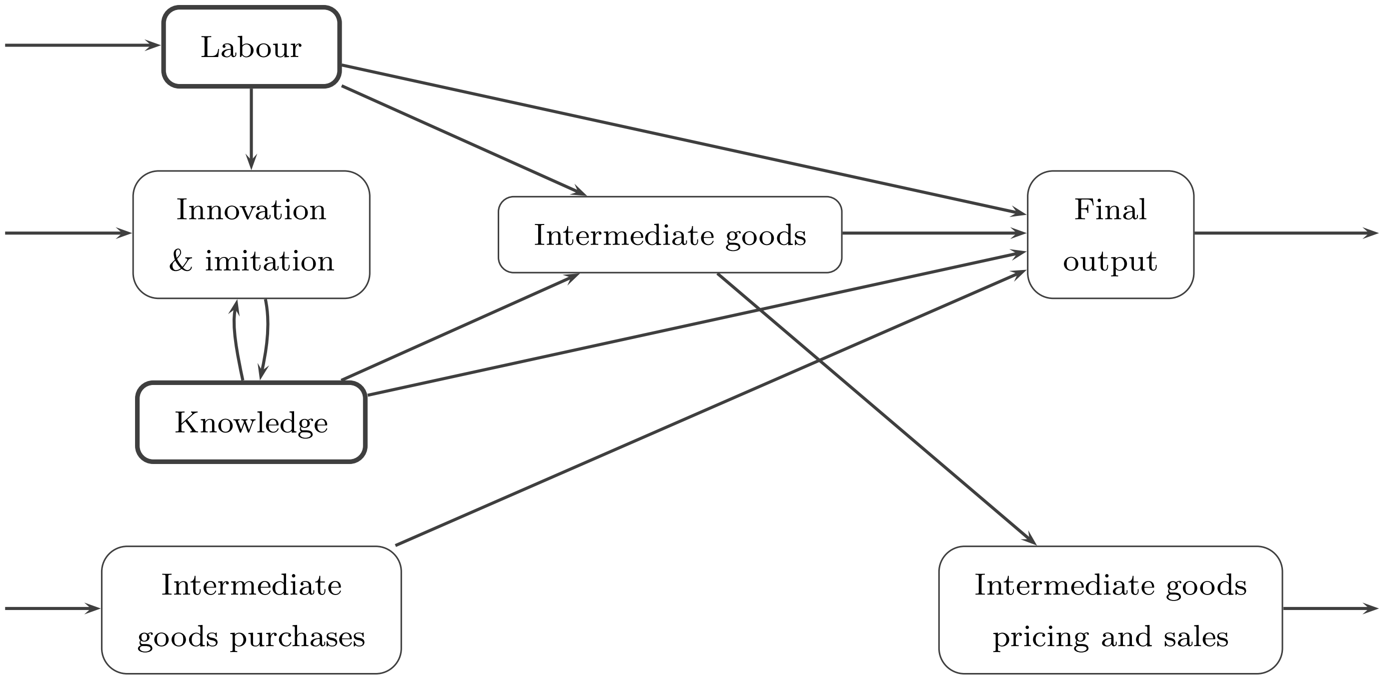


Figure 8: Structure diagram that only covers a single firm's activities in the *AL* Mark III models.

Two types of market in the *AL* Mark III models

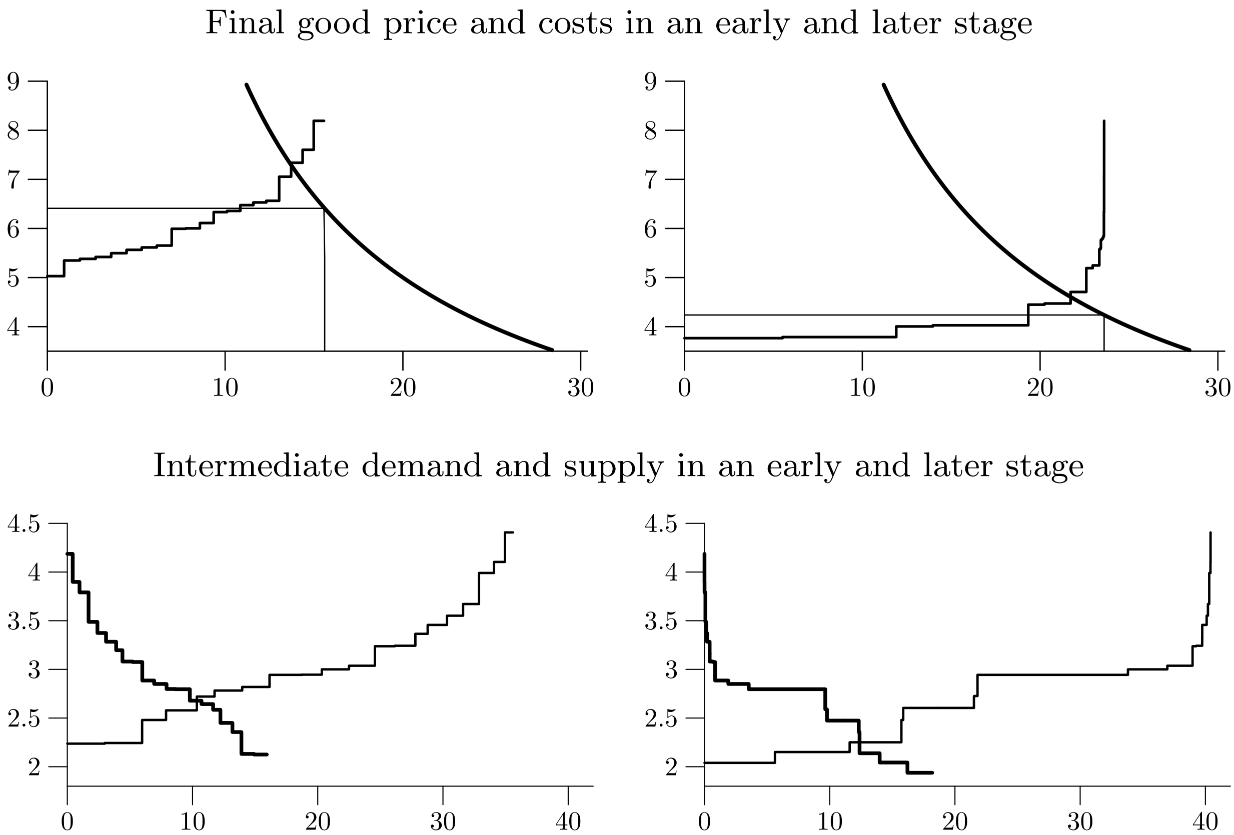


Figure 9: A comparison of between the final good market and a potential intermediate good market in the *AL* Mark III models

Two types of R&D strategy in the AL Mark III models

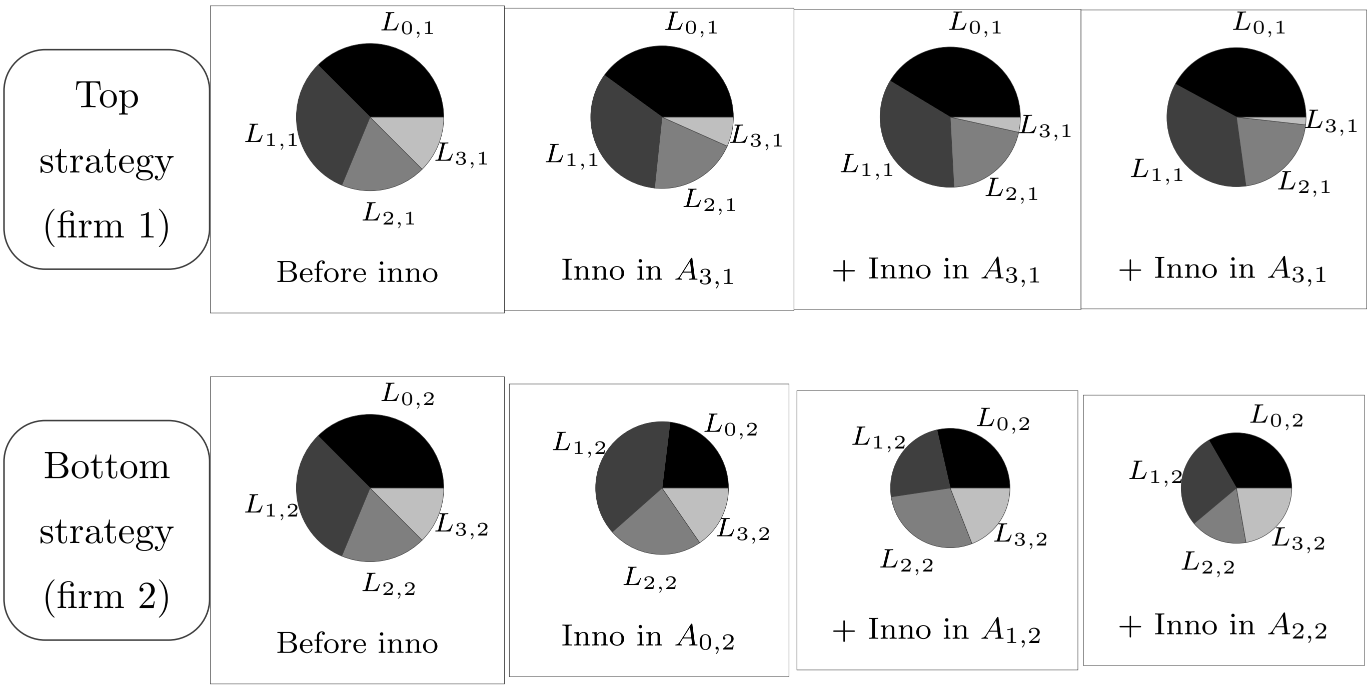


Figure 10: Comparison of two types of R&D strategy when there is no intermediate goods trade. The area of each pie is the labour costs of producing one unit of final output. The slices of the pie are the labour costs in individual activities.

Conclusions on the construction of the *AL* model family

The simple *AL* Mark I model with fixed productivities implies

- **A whole-economy approach** with very few parameters. This makes it easier to understand than the standard NW model
- **A pure-labour approach** that makes it easy to introduce e.g. fusions and fissions. This allows a simple solution to the entry and exit problem of NW models
- **A deductive approach** that gives proven theorems about the dynamics of the model
- **A simulation approach** that give a deeper intuition about the dynamics of the model and prepares for extensions
- **An *AL* model family** was not explicitly created. However, entry/exit of firms and the modelling of the productivity development of individual employees are such extensions

The *AL* Mark II models with changing productivities implies e.g.

- **The same approaches like for *AL* Mark I.** But we did not have time for that
- **Easy reduction to the *AL* model** by setting productivity change to zero. Thus we can check whether our simulation model is constructed correctly
- **Easy introduction of different regimes** of productivity change. Thus a whole family of *AL* Mark II models can be created quickly

The *AL* Mark III models are still described programmatically rather than in detail. However, we see e.g.

- **Intermediate goods rather than capital.** The models do not have to assume fixed prices of the produced inputs
- **Research specialisation** implies bindings on future possibilities, so it opens up a large area of strategy evolution
- **Transaction costs** can easily be introduced although they have been left out in the Nelson–Winter tradition (cf. Winter, 1991)
- **The theory of the firm** is closely related to multiactivity firms—and it is central to the verbal Nelson–Winter tradition (cf. Nelson and Winter, 1982, Part II)

Elements of the art of simulation

What can be said, can be said clearly, and what you can't say, you should shut up about—independently by Wittgenstein and the Computer

Basic principles for handling both evolutionary systems and the simulation of them

- **The KISS principle:** Keep It Simple, Stupid! Evolutionary modelling is so complex that it must be handled with care
- **The divide-and-govern principle:** Decompose hard tasks subtasks
- **The principle of successive approximations:** Start from the simple, and then move by tiny steps to the complex
- **The unite-and-lead principle:** Become a virtuoso with by means of the subtasks you have mastered

The levels of simulation work suggests a natural trajectory

- **Model application:** Explore a given model by a set of systematically performed simulations
- **Model development:** Extend given models or create a new one
- **System development:** Develop the tools for model development and application

Defining a simulation by four lists

1. **A list of model equations** like Quantity = Productivity ∗ Employment
2. **A list of parameter values** like Productivity = 1.2 (in the *AL* Mark I model)
3. **A list of initial values of state variables** like Initial_employment = 100
4. **A list of simulation run specification** like Simulation_steps = 5000

But . . . the computer needs more to perform a simulation run (e.g. provided by the Lsd system)

Working in loops

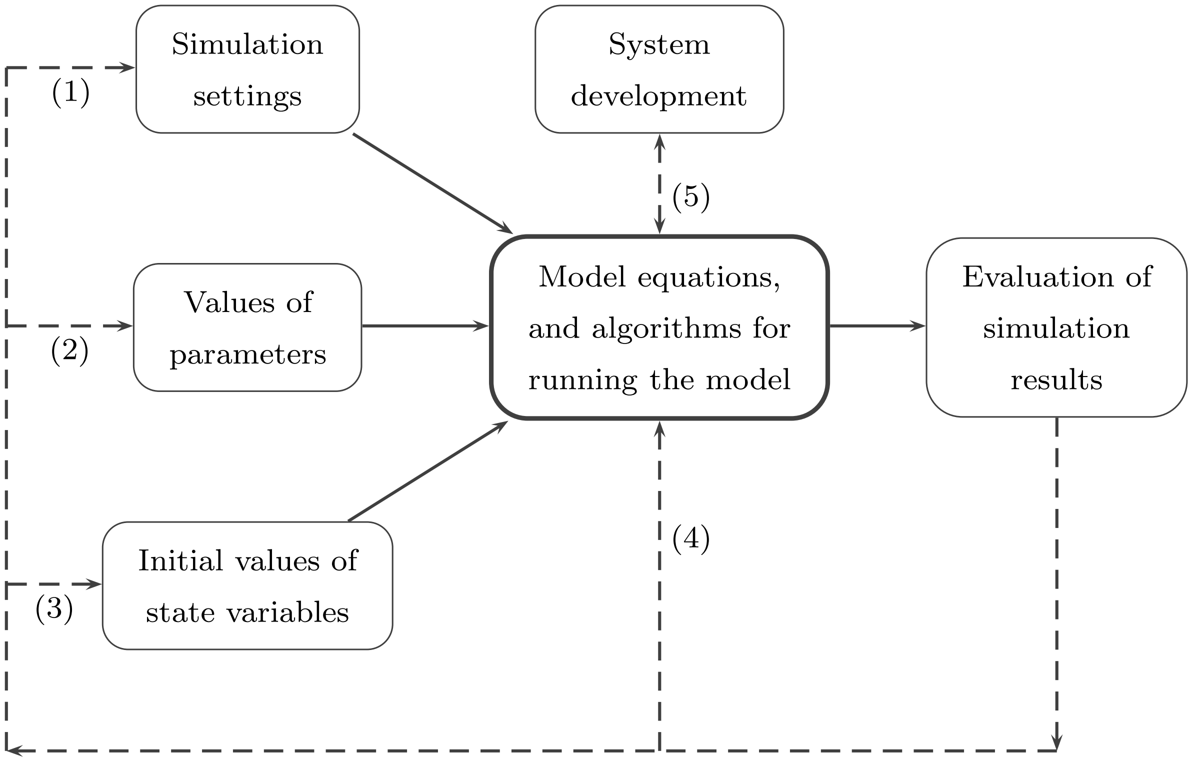


Figure 11: Feedbacks in the simulation of an evolutionary model.

Working in steps

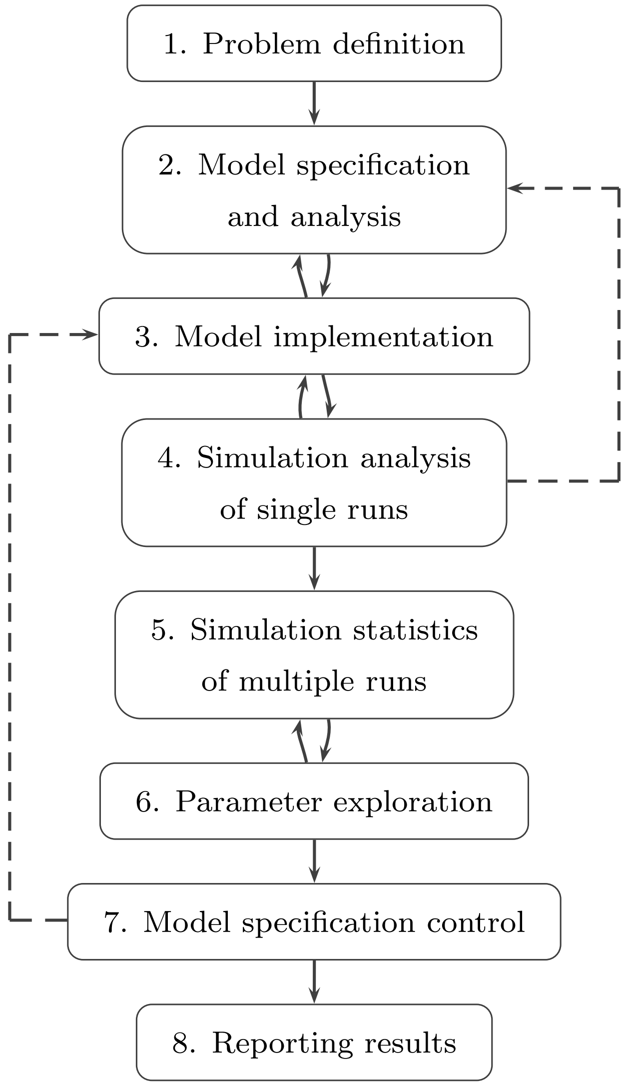


Figure 12: Steps in simulation work. A few feedbacks are included.

Laboratory for simulation development (Lsd)

The Lsd system is a system of simulation tools and evolutionary simulation models—see Andersen and Valente (2002) and Valente and Andersen (2002)

Purposes are

- **Facilitate exploration of existing models** by a windows-based interface to users that covers all the steps of model exploration
- **Facilitate extension of existing models** by a windows-based interface to developers that covers all the steps of model development and by a simplified interface to the C++ programming language
- **Facilitate creation of models** by the same methods as for existing models—but emphasising the bottom-up methodology
- **Facilitate documentation and replicability** of simulation results—thus, perhaps, overcoming the bad reputation of much simulation work
- **Facilitate the creation of a simulation community** by more-or-less enforcing rules for documentation and distribution of simulation models

Implementation of the system has been a continuing project by Valente since it was initiated in 1995 by Dosi, Nelson, Winter and others

- **The Lsd Models Manager (LMM)** is a computer application that is used for the selection, compilation and modification of stand-alone simulation models. For users interested only in model exploration LMM is a tool for choosing between preexisting simulation models. LMM is also used by the developer
- **The Lsd Model Explorers (LMEs)** are stand-alone applications for the simulation of models. The LME allows users to change the configuration and thus to perform loops 1–3 of figure 11 in an efficient manner. LME is also used by the developer

Working in steps in the Lsd system: Model users

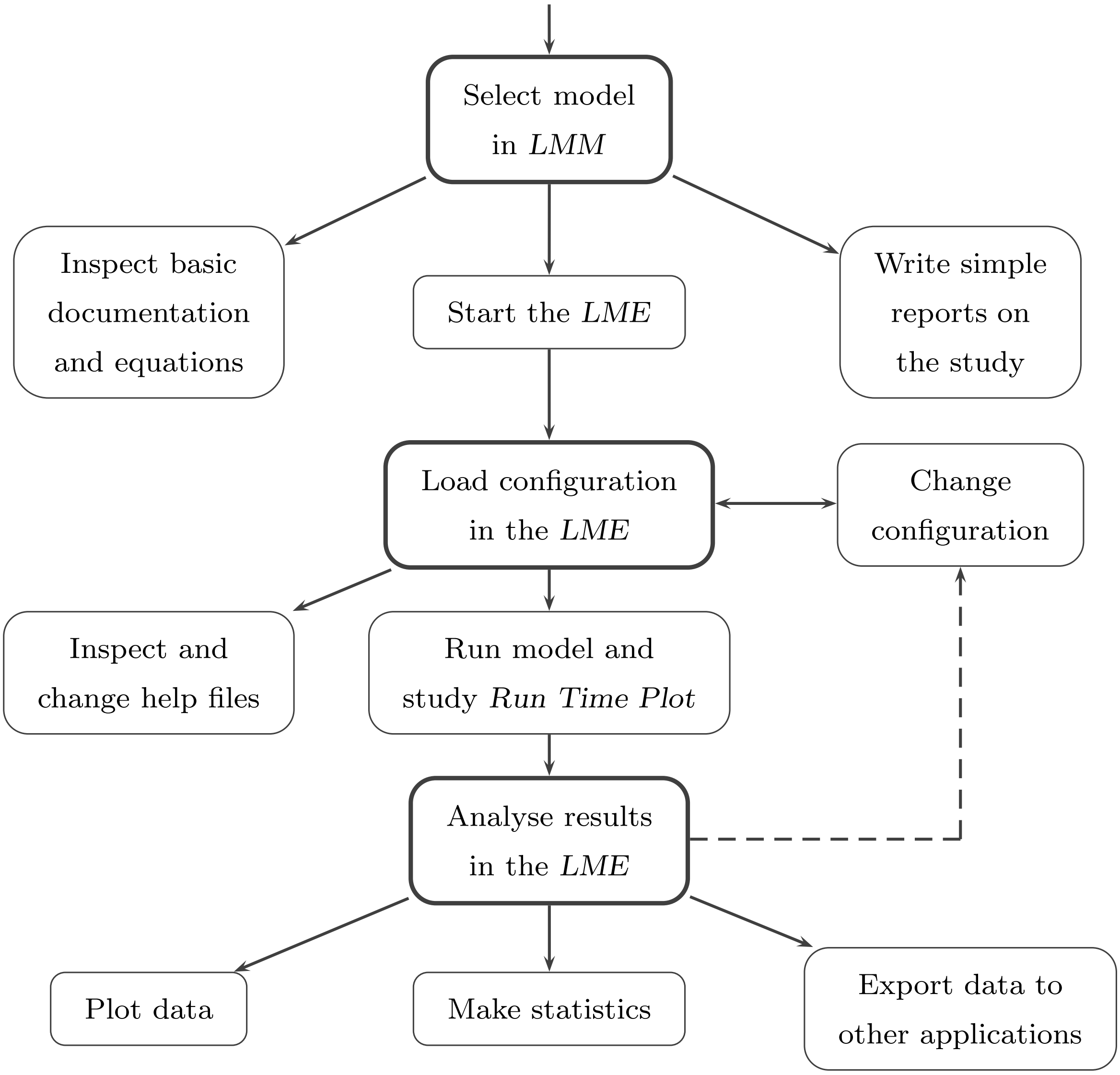


Figure 13: The basic use of the Lsd system.

First step: Select the model

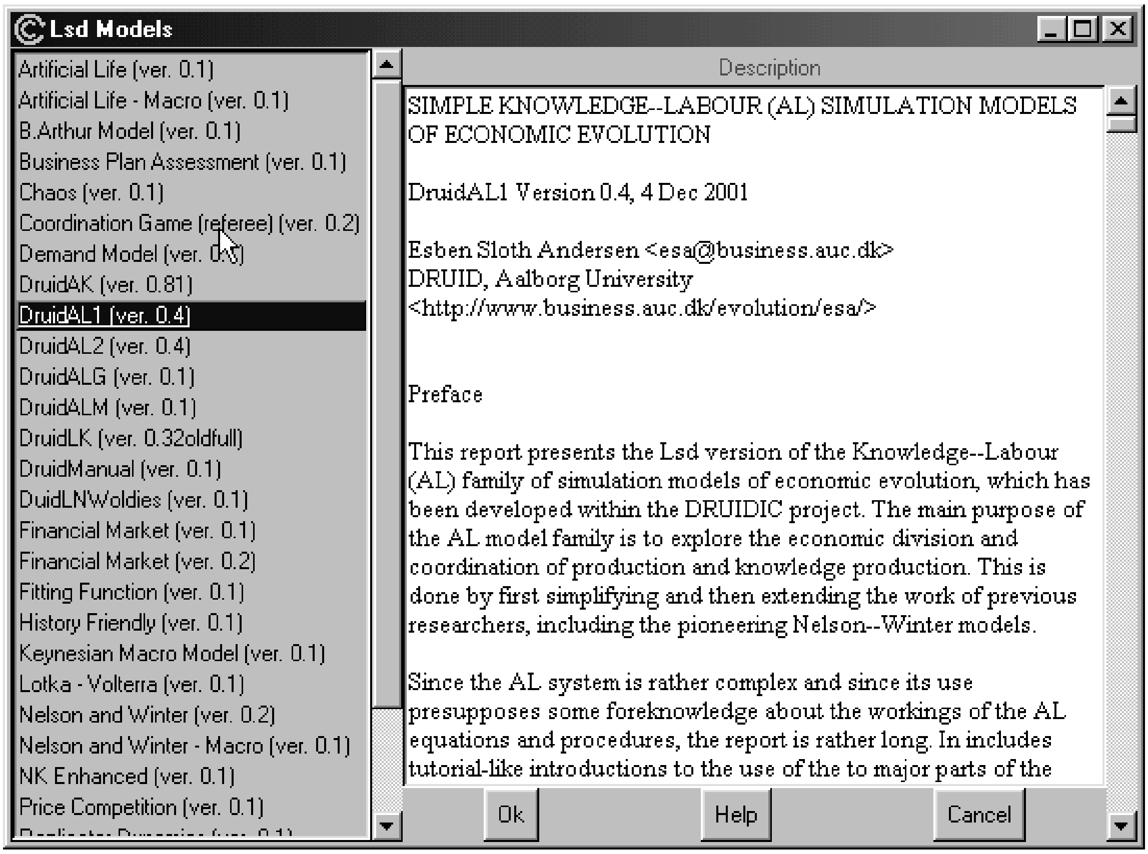


Figure 14: The window of the Lsd system that is first encountered by users. This window is the part of the Lsd Models Manager (*LMM*), where you choose the model with which to work. In the left part of the window there is a list of the models that are found in the Lsd directory of the computer. When a model is chosen, its description is shown in the right part of the window.

Working in Lsd steps: Start and configuration

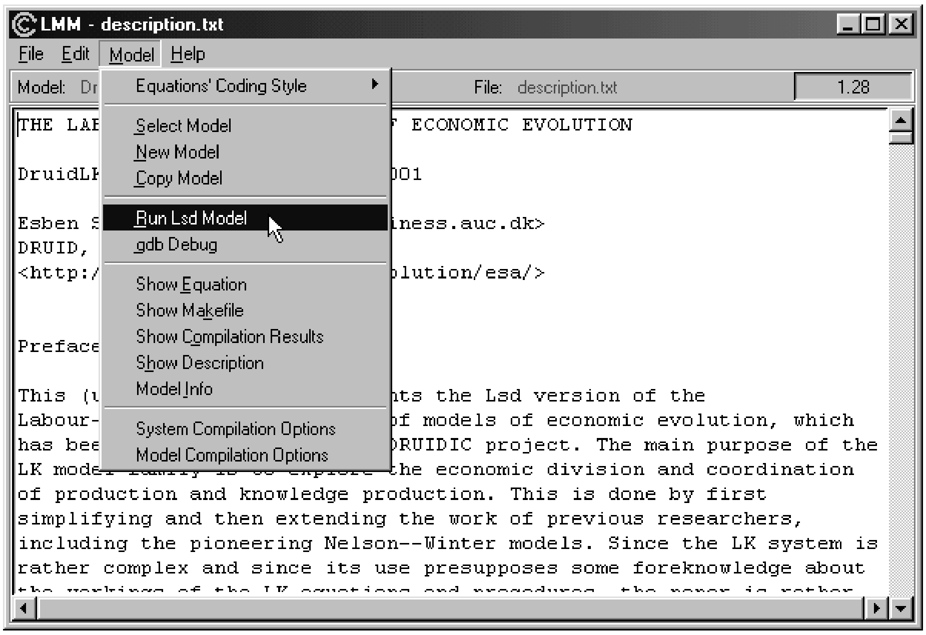


Figure 15: The main window of *LMM*. We have chosen to run the model immediately.

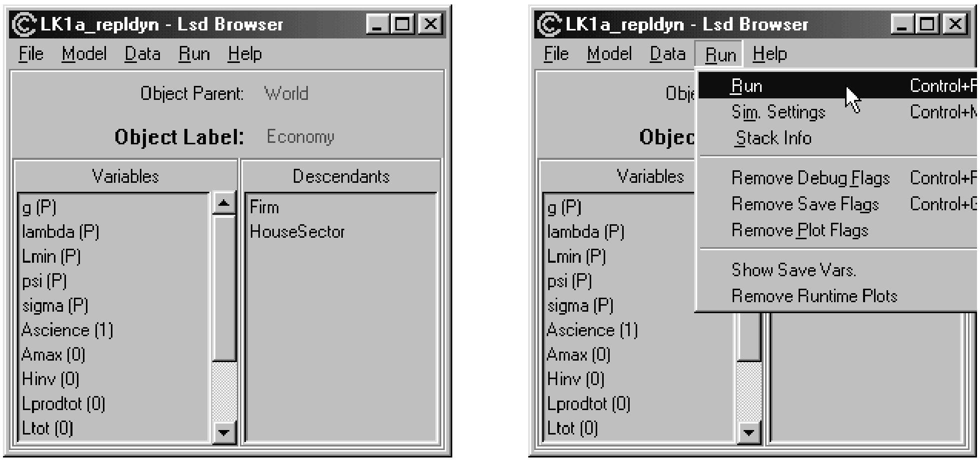


Figure 16: The browser window of the *LME* (Lsd Model Explorer) for an *AL*-like model has been opened by *LMM* and we have loaded a simulation specification called *LK1a_repldyn*. In the right panel the *Run* menu has been pulled down and we are ready to run the model.

Working in Lsd steps: Model structure and results

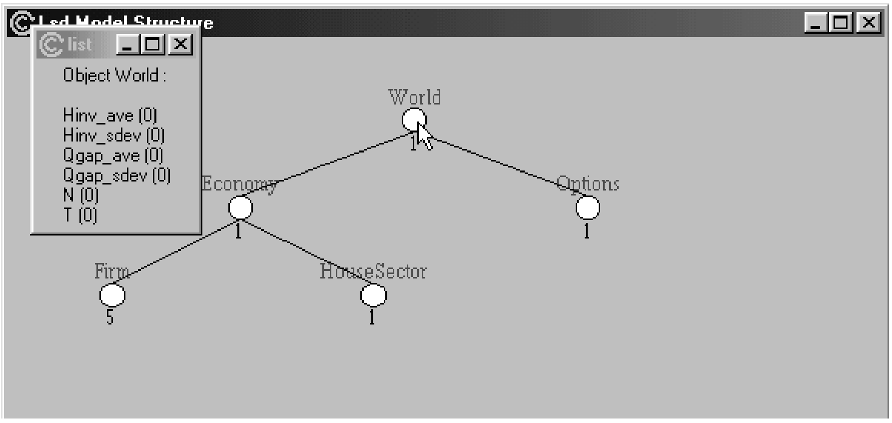


Figure 17: The model structure window of the *LME* for the *AL*-like model. There are five hierarchically organised objects: World, Economy, Firm, HouseSector and Options. Since the cursor is placed over the object World, we see the variables that are found here.

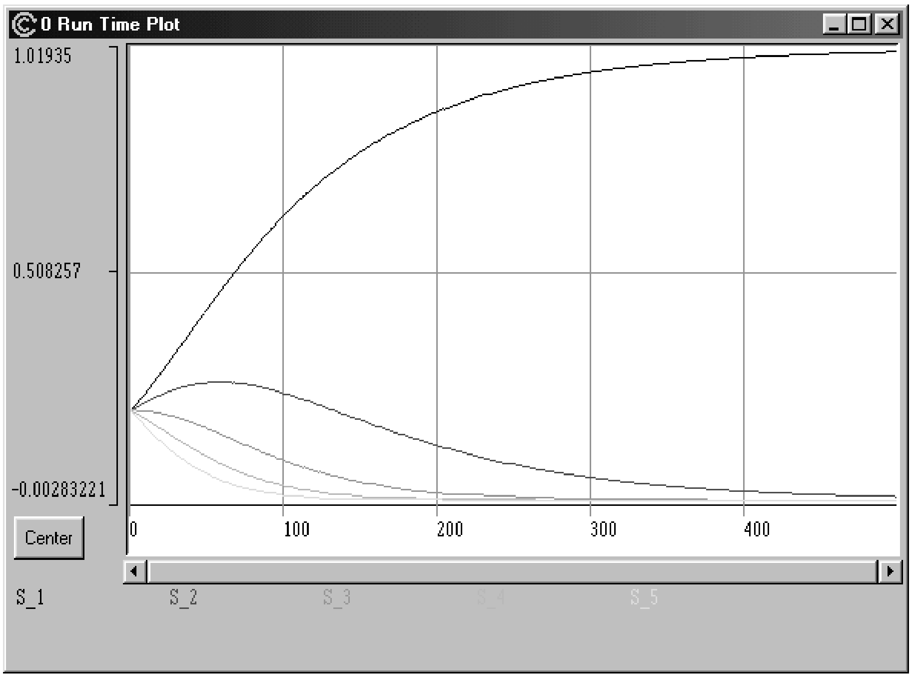


Figure 18: During a simulation run in an *LME* application it is possible to follow the dynamics of selected variables. The have selected the market shares of the five firms and see that they show a simple distance-from-mean dynamics. After the simulation run there are very rich possibilities for exploring and documenting the results. Here we can also produce more beautiful and precise graphics.

Working in Lsd steps: Model developers

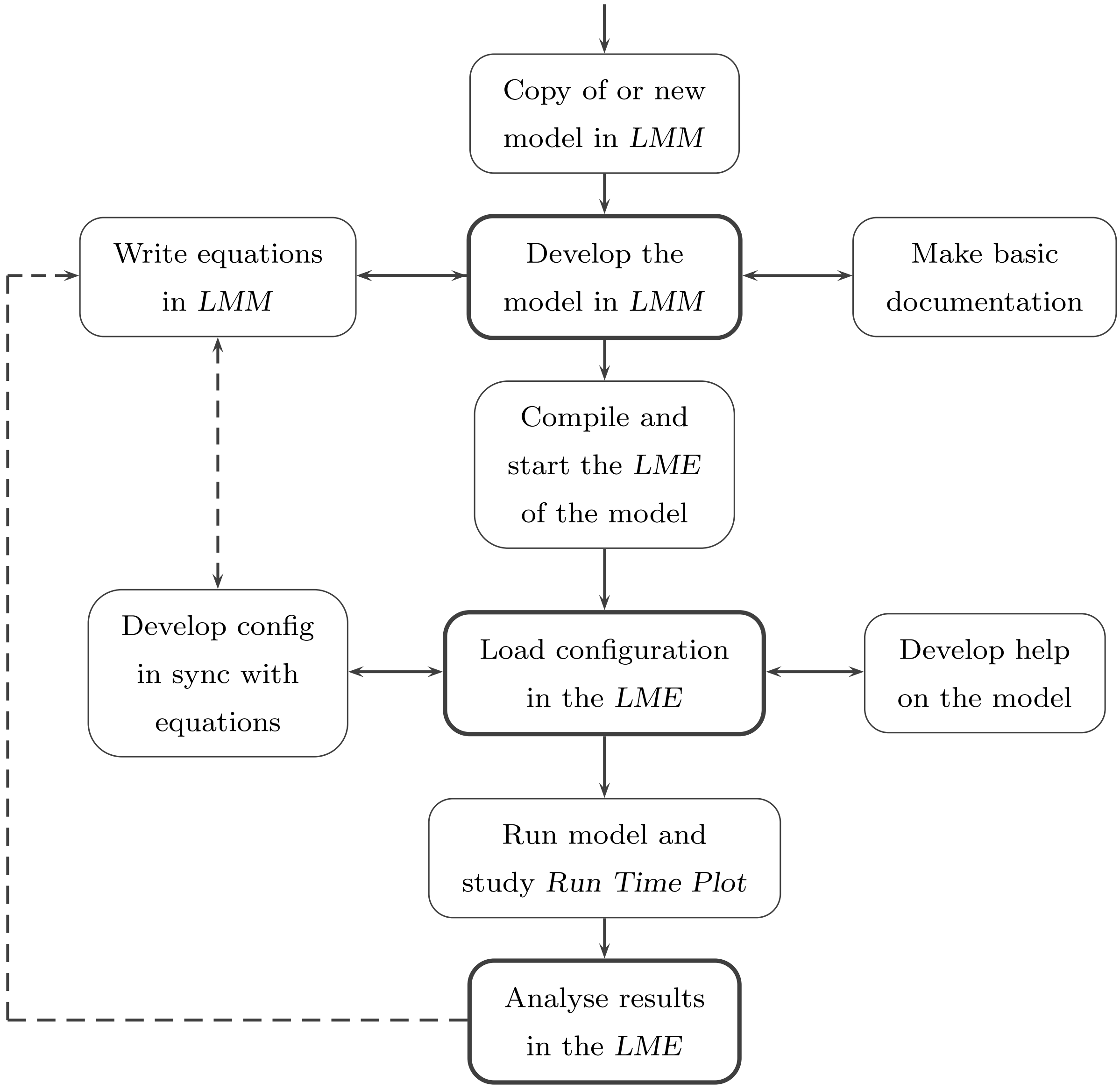


Figure 19: The development of Lsd models.

Working in Lsd steps: Writing equations

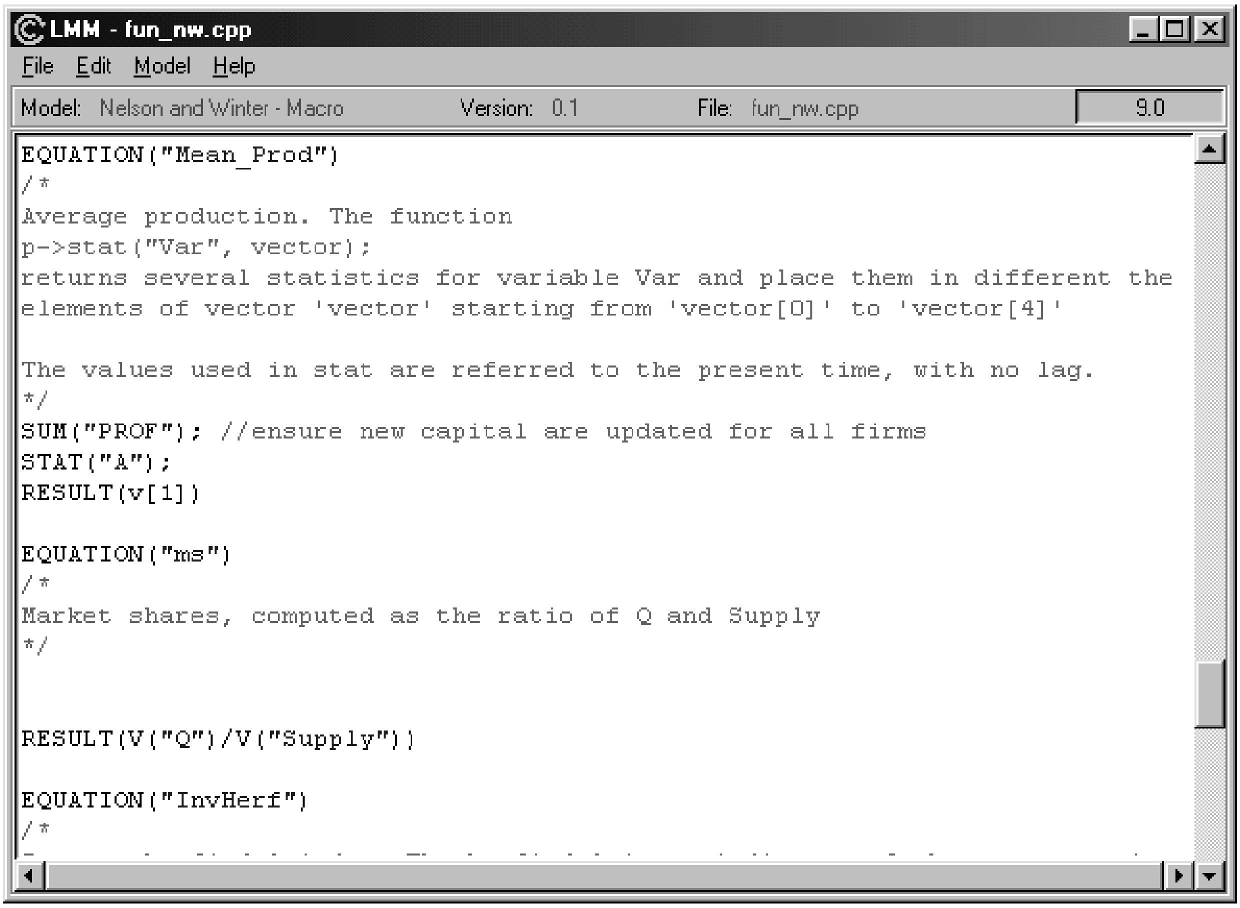


Figure 20: Here *LMM* is used for rewriting model equations. This can either be done in the a mixed Lsd/C++ language or (as here) in the highly simplified Lsd macro language. It is only the dark text that is macro code. The rest are comments.

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