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Toward a Multiactivity Generalisation of the Nelson–Winter Model

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Abstract This paper proposes a multiactivity generalisation of the Nelson–Winter model, or the NW model, in order to turn the attention of evolutionary minded economists toward specialisation and exchange, the emergence of markets for intermediate goods, the specialisation of R&D, and other issues of multisectoral growth and development. The argument and the solution is presented in four steps. First, there is a discussion of some practical difficulties and core theoretical problems in relation to the standard NW model of Schumpeterian competition. The conclusion is that this model gives an ad hoc solution to the tendency of evolutionary models to produce monopoly and that is has not really confronted the 'knife-edge' problems of the underlying Leontief technology. Thus there is still a need to confront what may be called the diversity paradox and the Leontief technology paradox of evolutionary modelling. Second, the paper develops a condensed version of the NW model that serves to highlight the theoretical problems and as a platform for the proposed generalisation. This version of the NW model, the LNW model, includes only labour and knowledge. With given technologies the LNW model shows standard replicator dynamics while it can also be used as a testbed for exploring different R&D regimes. Third, the bare bones of the multiactivity generalisation of the NW model, the MNW model, is presented. This presentation starts from firms that produce their intermediate goods by means of labour and knowledge. Then the focus turns to exchange in intermediate goods and the related problems of the specialisation of R&D. Fourth, the paper discusses the MNW model's partial solutions to the diversity paradox and the Leontief technology paradox as well as the possibilities of further developing and applying the MNW model.

Key words Evolutionary modelling – Nelson–Winter models – diversity paradox – Leontief technology paradox – labour-based evolution – multiactivity production – specialisation and exchange – markets for intermediate goods – specialisation of R&D – multisectoral growth and development

JEL-classification D2-L1-O3

1 Introduction: "history-friendliness" *and* generalisation in evolutionary modelling

Because of its pioneering status and its relative simplicity, the model of Schumpeterian competition presented in Nelson and Winter's *An Evolutionary Theory of Economic Change*[1] (1982, Chs. 12–14) has obtained a prominent role in defining what evolutionary economics is about. From this model we know that economic evolution can be depicted as a process in which firms follow rules or routines that can occasionally be mutated or adapted. We also know that an important example of economic evolution takes place within an industry (or an economy) where new process techniques are introduced and imitated. These and other aspects of the Nelson–Winter model have to some extent defined a 'paradigm' for further research on the conditions of R&D as a determinant of industrial concentration, dynamic competition in alternative technological regimes, the relationship between innovators and imitators, etc. A variant of the Nelson–Winter model (1982, Ch. 9) has also had some influence in promoting evolutionary growth theory.

The function of the Nelson–Winter model and the broader argument in the *Evolutionary Theory* has, perhaps, been most obvious in promoting and sharpening up the neo-Schumpeterian tradition of empirical research on technical change and industrial dynamics. But still there is quite a gap between the fairly abstract issues of their model and the concrete issues of the empirical researchers. It is, therefore, well-motivated when Malerba, Nelson, Orsenigo and Winter (1999, 3–6) argue that while the first generation of neo-Schumpeterian evolutionary economic models has largely been characterised by an attempt to understand the basic logic of evolutionary processes, the major challenge at the moment is to develop a second generation of "history-friendly" models that can be of major help for empirical research in economics. In other words, they suggest a second generation of specialised models after the first generation of generalised models. Thus it is not surprising that the "history-friendly" paper implicitly rejects the possibility of going directly for analytically solvable models. Instead the paper sets on to develop an evolutionary simulation model that reflects major stylised facts obtained by empirical researchers of the case of the computer industry, and it ends up not only by "history-replication" but also with "history-divergent" simulations. The results of the latter counterfactual simulations are intended to sharpen the research agenda for empirically oriented researchers of industrial dynamics, technological change, business organisation and strategy, etc. The immediate results of their paper suggest the need of emphasising the role of demand, the emergence of new technologies and markets, and the role of entry and venture capital.

Although the suggestion of a series of "history-friendly" models is to be much welcomed, the call for a second generation of models that is characterised *only* as "history-friendly" seems to be based on a one-sided diagnosis of the present state of neo-Schumpeterian modelling. It is correct "that the first generation of mostly very general evolutionary economic models has run into diminishing returns." (Malerba et al. 1999, 4) But from this fact does not necessarily follow that generalising work should be de-emphasised. The conclusion is rather that continued efforts to incrementally develop the evolutionary model are unlikely to give large increases in our knowledge about economic evolution, just like the Solow-model-based growth theory ran into stagnation and like the endogenous growth theory presently seems to be facing diminishing returns. But in such a situation a renewed emphasis on the specialisation of the models in relation to specific aspects of the evolutionary reality is only one of the adequate research strategies. The other main strategy is to study the premisses of the basic evolutionary model (here: the Nelson–Winter model) to see whether these premisses are only relevant for particular cases and whether a rather radical generalisation of the model might lead both to a widening of the area of empirical applications and to a new wave of work on formal modelling.

There are several theoretical, empirical and organisational reasons why a generalising strategy might be a fruitful complement to the specialising or "history-friendly" strategy. Among the theoretical reasons are first of all that the Nelson–Winter model only to a limited degree includes central aspects of its declared organisation-theoretic foundations. To be more specific, we see in Nelson and Winter (1982, Part II) account for the firm as an organisation a rich set of activities and decision-making problems but when we come to the model, the firm is reduced to a few decision problems centrered around one single production function and the related R&D. A generalisation to cover multiactivity firms and their related decision routines seems to be a core priority. Second, since the

Nelson–Winter model (even in its extensions) is basically of the one-sector type, it treats the emergence and evolution of markets in an ad hoc manner: it simply assumes a queue of potential entrants. An important generalisation is made possible if we instead start from innovative multiactivity firms; for these new markets may emerge when the firms exploit their productivity differentials through specialisation and exchange in intermediate goods. Finally, the Nelson–Winter model treats demand as an exogenous factor although it is obviously the central selective force in economic evolution. In the multiactivity generalisation the demand curves for intermediate goods emerge endogenously: they are simply constructed from the reservation prices that are in turn determined by the (evolving) activity-level productivities and the transaction cost level of the system.

The need for a multiactivity generalisation of the Nelson–Winter model does not only spring from the ambitions of theorists but also from empirically oriented modelling. Let us, to illustrate this point, consider the Malerba et al.'s (1999, 9–10) list of challenges to modelling that emerge from the stylised facts of the computer industry: (1) the shifting importance of different computer market segments, (2) vertical disintegration, and (3) the US dominance. As they are formulated, these challenges points to the addition of ad hoc features to the basic Nelson–Winter model. For instance, the addition of a two-dimensional evolution of computer characteristics (cheapness and performance) allows for the construction of different firm strategies concerning the segments. However, the empirical regularities of the computer industry may also be taken as a motivation for a generalisation of the Nelson–Winter model in order to make it more easy to cope with the computer industry and other industries. The multiactivity generalisation that is sketched above obviously facilitates the treatment of the issue of vertical integration, but it also—as will be shown later—suggests possibilities of handling the two other challenges. At the present point it, however, seems clear that the stylised facts of the evolution of not only the computer industry but also the software industry suggest a generalisation that treat explicitly the ever-changing division of labour within these industries and in their relationships to the rest of the economy.

Let us finally turn to the needs for a multiactivity generalisation that spring from the fragmented organisation of different kinds of research into economic evolution. These needs are implicitly dealt with by Dosi and Nelson (1994) and especially by Nelson's (1995) broad survey over empirically oriented evolutionary research from e.g. sociobiology and law via via growth and industrial dynamics to institutions. It is obvious that the areas of research have not only expanded greatly in recent years but that the research is also fragmented, perhaps to an increasing degree. There is little chance in the near future to overcome all this fragmentation, which is not only influenced by the different types of economists that have become engaged in evolutionary topics but also by contributions from other social scientist and biologists. This situation makes the fields of cultural evolution and business evolution and even the narrower field of studies of basic economic evolution rather confusing, not only for outsiders but also for many of the researchers. Although the confusion is not likely to be overcome in the near future, attempts to generalise e.g. the Nelson–Winter model may help to promote the dialogue between the different researchers and, hopefully, to improve the situation.[2]

If the specialisation of the Nelson–Winter model as a complement needs a major generalisation, the problem is why generalisations hitherto have mostly been in the form of limited extensions, what the major characteristics of the generalisation should be, and how a concrete generalisation would look like. The paper provides an argument on these issues that leads to a generalised Nelson–Winter model with multiactivity firms, innovative specialisation, and a multisectoral economy with exchange not only in the final good but also in intermediate goods. The argument and the solution is presented in four steps. The first step is made in section 2 that presents the task of generalising the Nelson–Winter model (abbreviated as the NW model) and the related problems, where 'problems' are taken in a double sense: the practical difficulties and the core theoretical problems that are dealt with (the diversity paradox and the Leontief technology paradox). The second step is in section 3 to develop a condensed version of the Nelson–Winter model that serves to highlight the theoretical problems; this is done be means of a model that includes only labour and knowledge—the LNW model. Section 4 deals with third step by presenting the bare bones of the multiactivity generalisation of the Nelson–Winter model—the MNW model—and by studying the dynamics of this model. The fourth step is to discuss and develop further the multiactivity generalisation in the last parts of section 4.3. Finally, section 5 draws some conclusions.

2 The Nelson–Winter model (NW) and its paradoxes

2.1 The strategy of 'tradition-friendly' generalisation

The basic idea behind the generalising strategy of the present paper is that many difficulties of the Nelson–Winter tradition are to a significant extent the result of the lack of generality of the basic Nelson–Winter model. This means that the Nelson–Winter model only is fruitful for the analysis of a particular set of evolutionary economic phenomena. The main solution is thus to make a generalised model that, more or less smoothly, includes the existing model as a special case. Since generalised models easily loose their connection to empirical work, the generalising task has to be further specified. Especially it should be emphasised that the generalisation should lead to an increased area of potential interaction with empirical work.[3] Another priority is to obtain an increased area of interaction with other traditions of evolutionary analysis as well as with theorists of endogenous growth and trade.

Before we turn to the generalisation itself, a couple of things about the strategy of generalisation should be noted. These caveats will be put in a rather general form, so that we don't enter prematurely into the details of the suggested 'generalisation' of the Nelson–Winter model. An important caveat is that for a complex model generalisation is also abstraction from many of the details in the original model. Let us, to see what this means, for a moment consider models as consumer goods and naively interpret generalisation in a purely positive manner. Then we in the Lancaster (1979) framework are apparently allowed to say that a generalisation corresponds to a vertical product differentiation while a specialisation corresponds to a horizontal differentiation. If this was the case, a generalisation implies that the generalised model is better than the original for at least one characteristic and at least just as good in all other characteristics. But this is not the case for generalisations. One problem is the loss of details because of generalisation. The reason is that the general theory often has to ignore many of the special assumptions that characterised and enriched the special theory—so it becomes more abstract. Furthermore, the use of more generally applicable assumptions will often lead to a refocussing of the theory.[4] So, in fact, we should compare generalisation with horizontal product differentiation.[5] To put it more concretely, the original Nelson–Winter model and its immediate extensions are good for some purposes that the generalised model treats in a simplified way in order to expand radically the range of potential applications.

Another thing to note is that generalisation springs out of the difficulties in applying existing theories and models—as it is seen in the historical sequence of generalisations of mathematical number theory to include negative numbers, rational numbers, real numbers, and complex numbers.[6] Each step becomes understandable when we study the problems of the previous historical version of the theory. In the case of a complex model like the Nelson–Winter model things get more complicated, but the logic of generalisation is the same: find basic problems in the model and try to solve them by means of generalisation. In the following we shall see that there are at least two basic problems related to the Nelson–Winter model. The first problem is that the tendency of evolutionary processes to produce uniformity of behaviour and monopoly in the Nelson–Winter model is countervailed by what from a larger perspective seems to be an ad hoc solution. The second problem is that the model's assumption that production is an outcome of several complementary inputs (Leontief technology) leads to radical inflexibility that are resolved only for a very special case (the innovative substitution between capital and labour). These problems suggest a certain direction in the generalisation of the Nelson–Winter model. But the Nelson–Winter model has, of course, other explicit and implicit ambitions and problems that call for a solution. Since no process of generalisation can solve all problems, the choice of the set of problems to focus upon is quite important for the direction of the generalising efforts. There are many possible alternatives to the choice made in the present paper.

Having put forward these caveats, it should also be remembered that even 'tradition-friendly' generalisation is a basic research strategy that often leads to interesting and even surprising results as we see when comparing the Solow growth model with the endogenous growth models. The question is, of course, whether this is also the case for the present multiactivity generalisation of the Nelson–Winter model.

Box 1 Nelson–Winter models in the narrow sense.

Even in a narrow sense the 'Nelson–Winter model' is really a family of closely related models that are applied in or directly descendent from Nelson and Winter (1982). The core NW models are relatively complex formal models that have been implemented for computer simulation (see the second and third part of the listing below[7]). To this core are connected more simplified models (see the first part of the listing below). These simpler models are designed to provide results through formal mathematical analysis and thus to elucidate certain aspects of the behaviour of the more complex models. To make it easy to refer to the models, the following naming system is applied: All models are called NW with an added postfix. For models in the *Evolutionary Theory*, the postfix is simply the number of the chapter or the section in which the model is described. For later publications NW is prefixed with an X (for eXtra or eXtended) and postfixed with the two last digits of the year of publication. In the listing there is also mentioned some of the background papers for the models of the book.

NW6 "A Particular Model of Economic Selection": Nelson and Winter (1982, Ch. 6, 144–154); see also Winter (1964, 1971).

NW7 "A Markov Model of Factor Substitution": Nelson and Winter (1982, Ch. 7, 175–192); see also Nelson and Winter (1975).

NW10.1 "Development and Backwardness in a Two-Technology Evolutionary Model": Nelson and Winter (1982, Ch. 10, 235–240, 240–245); see also Nelson (1968).

NW10.2 "Growth as a Pure Selection Process: Many Techniques and Many Variable Inputs": Nelson and Winter (1982, Ch. 10, 240–245).

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NW9 "An Evolutionary Model of Economic Growth": Nelson and Winter (1982, Ch. 9, 209–214); see also Nelson, Winter and Schuette (1976).

NW12 "Dynamic Competition and Technical Progress": Nelson and Winter (1982, Ch. 12, 281–287, 302 f.); see also Nelson and Winter (1977).

NW13 "Forces Generating and Limiting Concentration under Schumpeterian Competition": Nelson and Winter (1982, Ch. 13); see also Nelson and Winter (1978).

NW14 "The Schumpeterian Trade-off Revisited": Nelson and Winter (1982, Ch. 14); see also Nelson and Winter (1982b).

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XNW84 "Schumpeterian Competition in Alternative Technological Regimes": Winter (1984).

XNW99 " 'History-friendly' Models of Industry Evolution: The Computer Industry": Malerba, Nelson, Orsenigo and Winter (1999).

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Further works: Neither analytical NW models published after 1982 nor the contributions of other authors are listed (the latter are partly surveyed in Kwasnicki 2001).

2.2 The design of the Nelson–Winter model and the diversity paradox

Let us now turn to the Nelson–Winter model, which it for shortness is often convenient to call the NW model or simply NW. The first problem we meet is that when talking of a single Nelson–Winter model, a single NW, we are making an abstraction from the rich reality of the *Evolutionary Theory*. In this book we find that Nelson and Winter have in fact produced not one but a whole family of more or less related NW models. These models are listed in box 1. This list of models demonstrates that then NW family has multiple purposes spanning from contributions to general selection theory and growth theory to more concrete issues of economic development and Schumpeterian-style industrial dynamics. For many of these purposes particular versions of NW have been produced. So when we talk of 'the' NW model, we are operating at a rather high level of abstraction. There is, on the other hand, a basic family likeness between the NW models. One important task is to formulate what this likeness is about and how it may be integrated in a more general class of models.

With respect to likeness we quickly recognise that the NW models are specialised members of a very general and flexible family of discrete-time Markov models. Such models describe a dynamic system whose state variables at time t determine the state variables at time $t+1$ through a set of transition rules that at least partially are of a stochastic nature. The family of NW models is a subset of such Markov models where the state variables are interpreted as economically relevant stocks like

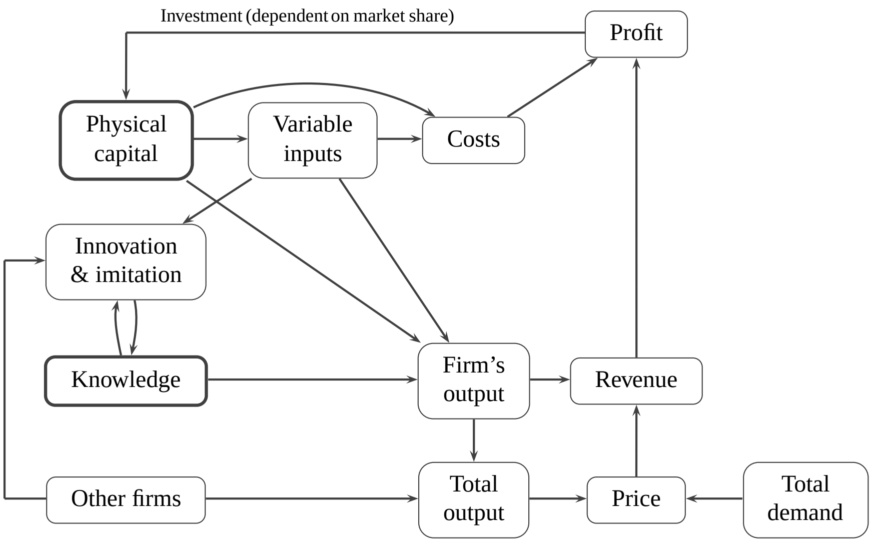

Fig. 1 Structure diagram[9] of the standard Nelson–Winter model of industrial dynamics. The diagram puts an emphasis on a particular firm, while aggregates are placed in the last row. An arrow from *x* to *y* should be read '*x* codetermines *y*'.

the firms' capital and productivities (based on knowledge). Similarly, the transition rules are formulated in economically relevant ways. For instance, there are rules about (A) how variable inputs are hired and combined with capital and knowledge to produce output and generate profits in a deterministic way, (B) how other variable inputs are used for research activities that improves knowledge in a stochastic way, and (C) how the stock of capital is transformed by deterministic rules based on profitability and market shares. Since such an NW model is still a Markov model, the rich set of theorems of such models can in principle be applied to the NW model. However, to allow for analytical solutions a radical simplification and redesign of NW model are normally necessary as we see it in the simplified theoretical models of the *Evolutionary Theory* (in box 1 are called NW6 and NW7) as well as in the more complex but still highly stylised models that lead to the theorems of Winter, Kaniovski and Dosi (2000).

When the present paper talks about the need for a generalisation of the NW model, the task is obviously not to move the whole discussion back to the austere level of Markov models or models of other stochastic processes. But the more complex NW models of box 1, i.e. NW9, NW12, NW13, NW14, XNW84 and XNW99, have some common strategies in their specification of the state variables and the transition rules that give them a close family likeness. This likeness makes it possible to take on of the complex models at a representative NW model or *the* NW model. The present author (Andersen 1996b, Ch. 4) has in another context suggested that NW12 fulfils this function. But when we are dealing with the issue of generalisation this is not sufficient since there will e.g. also be a need of referring to the NW growth model (NW9). At a more general level there will even be a need to make inferences about the class of 'obvious' extensions of the NW framework, or the "vast array of particular models can be constructed within the broad limits of the theoretical schema" of Nelson and Winter (1982, 19). Therefore, it is necessary to give a stylised account for the backbones of the standard NW model. As our standard NW model we shall not take the NW growth model (since NW9 lacks essential features like costly R&D) but instead the NW model of industrial dynamics. The main elements of this model (NW12, etc.) is depicted in figure 1.[8]

Since Nelson and Winter adhere to the 'dynamics first' principle, the quickest way to describe the NW model is to consider how its dynamics is determined by three processes: (I) the selection process, (II) the innovation–imitation process, and (III) the process of capacity accumulation. Let us consider them in turn:

I. *The selection process* emerges from the fact that in the NW model all firms face the same price although they may have different unit costs. These differences are based on different capital productivities that imply that a given stock of capital (and thus a given cost level) can produce different quantities in different firms. Thus the differences lead to different profit rates. We may also say that firms are being ‘selected’ in the sense of being given different profit rates; supernormal profits can be considered as rewards for high fitness, while subnormal profits are punishments for low fitness. If the bottomline profits are directly transformed into expansion or contraction and if the firms have different productivities, we have a case of pure *replicator dynamics*. Here the best performing firm in a deterministic way will ultimately become the sole supplier of output. The analytic NW models (NW6, NW7, NW10.1, and NW10.2) show variations over this theme.

II. *The innovation–imitation process* both produces and controls the variance of the firms’ productivities. Since fitness variance is of central importance, the complex NW models show great ingenuity in the specification of the innovation–imitation process. One of them (NW9) considers this “search” process as an ad hoc activity that emerges costlessly when a firm experiences unsatisfactory results, but the rest of the NW models have R&D as a permanent activity. These models (NW12, NW13, NW14, XNW84, XNW99) may, in this respect, be considered to represent the standard NW model. This standard model demonstrates that, as soon as R&D is introduced, the innovation–imitation process does not remove the long-term tendency toward monopoly. The reason is that although R&D activities show constant returns with respect to change of productivities, there are increasing returns to the application of their results. This means that, compared to a small firm, a large firm will not only have more researchers but also larger returns from each R&D result. Therefore, if firms show no restraint on their expansion, there is a probability of one that one of them will ultimately become the sole supplier.

III. *The process of capacity accumulation* has already been touched upon in relation to I and II, where an emphasis was put on the standard NW model’s tendency to produce monopoly. For analytical and empirical reasons it is important to include into the model some countervailing forces. In the standard NW model the solution is to introduce a monopolistic restraint on capacity accumulation. In the tradition of Cournot oligopoly analysis the restraint is modelled as the firm’s increasing awareness of the overall demand curve as its market share increases. In the NW context a large firm, of course, also have to take into account the potential productivity change of its competitors, but nevertheless the propensity to accumulate will decrease as the market share increases; there will be zero accumulation before monopoly is reached. In other words, the process of concentration stops before monopoly is reached. (The decision rules that lead to this result are described in the extensive endnote 10 for easy reference.[10]). An alternative strategy to control the tendency toward monopoly is to introduce new firms into the industry. These new firms enter from an exogenous pool of firms (cf. e.g. Winter 1984, 283–288).

When we consider the three processes that characterise the standard NW model, it should be noted that the innovation–imitation process (II) is most developed, while the selection process (I) takes place in a simplistic environment (a simple market for a homogeneous good) and that the capital accumulation process (III) is characterised by a a special version of monopolistic restraint. The lack of generality of the two latter processes hangs together. In the simplistic selection environment the trend toward uniformity and monopoly is strong (especially since it is supported by the characteristics of R&D). Therefore, we clearly see what may be called the *diversity paradox* of evolutionary analysis. In its basic form the paradox is that we in empirical reality see a large degree of diversity of behaviour although there is a very strong tendency of evolutionary models to remove this diversity. The diversity paradox can also be formulated more narrowly as the non-monopoly paradox that in empirical reality we see so few examples of full-blown monopolies although evolutionary models suggest a strong tendency toward such monopolies. The NW model’s specification of the process of capital accumulation has, to some extent, the character of an ad hoc solution to the non-monopoly paradox. To put it crudely, the solution is basically to say that we see no monopolies because firms really do not want to become monopolies under the given demand schedule.

The trend toward uniformity and monopoly in the NW model is, as already mentioned, largely an outcome of its highly simplified selection environment. Thus a general solution of the diversity paradox will have to introduce a more complex selection environment. Such a solution will probably, as emphasised by Romer (1993, 556), have some of the characteristics of Chamberlin’s (1962[1933]) model of monopolistic competition, where firms are facing a much more complex selection environment that in the standard NW model. The “history-friendly” model of Malerba et al. (1999) can, to some extent, be seen as a response to this challenge, and there has also been been several attempts to construct a more general selection environment by studying the evolution of demand (see e.g. Aversi, Dosi, Fagiolo, Meacci and Olivetti 1999). These attempts have, however, not been developing the intricsic logic of the NW model. For this purpose it seems relevant to start from a simplified version of the NW model that does not presuppose a very special solution (we shall see how in section 3). From this starting point we can consider the introduction of simple model characteristics that, more or less, overcome the trend toward monopoly. To do so is a core challenge for the generalisation of the NW model.

2.3 The notion of evolving Leontief technology and the paradox of Leontief technology

Nelson and Winter started their collaboration with the publication of a series of papers on evolutionary growth theory and related Nelson–Winter models (see NW9, NW10.1 and NW10.2 in box 1; see also Nelson and Winter 1973, 1974), and this starting point has influenced the basic design of the NW models. Therefore, we shall for a while be more concrete and consider the character of this influence. At the time in the early 1970s when the NW model was emerging, standard economic growth theory had almost forgotten the original Harrod–Domar model and were instead engaged in the development of Solow-like models and their application to empirical growth accounting. In this situation Nelson and Winter more or less directly said that the production functions (of the Cobb–Douglas family) that are underlying Solow-like models are really unwarrented modelling tricks and that growth theory should, in an evolutionary way, return to the kind of technology that is underlying the Harrod–Domar model (cf. Nelson and Winter 1982, Ch. 8). They added to the sense of provocation by constructing their evolutionary alternative to standard growth theory as an NW computer simulation model (cf. Nelson and Winter 1982, Ch. 9). But, in retrospect, it may be said that Nelson and Winter were only provocative to economists who were not willing to follow the logic underlying their contribution.

The Harrod–Domar model is based on a aggregate version of what is normally called the Leontief (1966[1941]) production function, where there are constant returns to scale and where physical capital and labour have to be used in a fixed proportion. The infamous Harrod–Domar ‘knife-edge’ problem emerges from the macroeconomic use of this production function in which the employment of the aggregate capital K is constrained by the aggregate labour L, and vice versa. This constraint functions via the productivity of capital A^{K} and the productivity of labour A^{L}. According to its own productivity capital could produce an output of $Q^{\mathrm{K}} = A^{\mathrm{K}}K$ and similarly labour could produce $Q^{\mathrm{L}} = A^{\mathrm{K}}L$. But in both cases it is presupposed that the complementary amount of the other factor is supplied. Thus we have the actual output as $Q = \min(Q^{\mathrm{K}}, Q^{\mathrm{L}})$. If $Q^{\mathrm{K}} < Q^{\mathrm{L}}$, capital is the constraining factor and the economy experiences unemployment because only a part of the available labour is used, namely $\frac{A^{\mathrm{K}}}{A^{\mathrm{L}}}K$. If $Q^{\mathrm{K}} > Q^{\mathrm{L}}$, labour is the constraining factor and this means that only a part of the available capital is used, $\frac{A^{\mathrm{L}}}{A^{\mathrm{K}}}L$. In this setting the Harrod–Domar model furthermore assumes a fixed savings rate, a fixed capital depreciation, and a fixed growth rate of the labour force. So it is not surprising that the model leads to a prediction of catastrophe: continually increasing unemployment or continually decreasing utilisation of the stock of capital. Such a radical knife-edge problem is, of course, highly implausible. It is an artefact of the use in the Harrod–Domar model of too many exogenous parameters.

Nelson and Winter (1982, 196) obviously consider the Harrod–Domar model a poor tool for thinking about the stylised facts of economic growth, but they disagree with the standard solution of exchanging the Leontief production function with a production function that allows for a smooth and instantaneous substitution between capital and labour. Instead they base their model on what may be called evolving Leontief technology. Such a technology can only be studied from the micro level, while the aggregates studied by standard theory are just aggregates that—except in very special and unlikely cases—are composed of firms that differ with respect to their productivity levels. Since the NW model applies Leontief technology, each firm j in period t has to cope with the fact that the employment of the available capital K_{jt} is constrained by the available labour L_{jt}, and vice versa.

However, if one of the inputs (normally L_{jt}) is variable in the short run, the cost minimising solution is, of course, only to employ the quantity of labour that can be productivity used. This means that the firm calculates from the Leontief function $Q_{jt} = \min(A^{\mathrm{K}}_{jt}K_{jt}, A^{\mathrm{L}}_{jt}L_{jt})$ how much labour to hire, i.e. $L_{jt} = A^{\mathrm{K}}_{jt}K_{jt}/A^{\mathrm{L}}_{jt}$. The firm's reaction to a changed environment like an increase in the relative costs of labour is to redirect the employed researchers L^{res}_{jt} for an R&D effort that at best influences the situation in period $t+1$; for this period there is an increased probability that $A^{\mathrm{L}}_{j,t+1}$ is improved.[11] This possibility leads the NW model to conclusions that are quite different from those of the Harrod–Domar model: innovation supplies part of the longer-term flexibility that is needed to avoid the radical knife-edge problem in an economy based on Leontief technology.

The Nelson–Winter solution to the knife-edge problem of Leontief technology is, however, quite limited as they also implicitly recognises. The problem is that Leontief technology implies that output is not only produced by a fixed proportion of capital and labour but that all the intermediate goods are also used in fixed proportions. In a complex economy this means that there is a whole nightmare of knife-edges to be taken into account, as it will be recognised by anyone who have contemplated about the implications of literal reading of an input–output table. [12] The NW model circumvent the problem by focussing only on the industry that supplies the final output and by assuming that the intermediate goods industries are able to deliver the required quantities. For the final goods industry Nelson and Winter (1982, e.g. 144, 176, 240, 281, but not NW9) normally assume Leontief technology with respect to all inputs. Thus they apply a full Leontief production function that should really be written as

$$Q_{jt} = \min(\underbrace{A^{\mathrm{K}}_{jt}K_{jt}, A^{\mathrm{L}}_{jt}L_{jt}}_{K \text{ and } L}, \underbrace{A^{\mathrm{X}}_{1jt}X_{1jt}, \ldots, A^{\mathrm{X}}_{mjt}X_{mjt}}_{m \text{ intermediate goods}}), \tag{1}$$

where X_{ijt} is the quantity of the ith of the m intermediate goods and A^{X}_{ijt} is the productivity of that intermediate good, i.e. the amount output produced per unit of the intermediate good. Each of these productivities could evolve both in general or as a response to changes in relative input prices. But since the NW has shown how this works for capital and labour, the fairly easy generalisation is not dealt with. Instead the prices and productivities for the intermediate goods are frozon so that they can be aggregated to an unchanging cost per unit of capital. In the NW models of Schumpeterian competion (NW12, NW13, NW14, XNW84) labour is treated similarly, so it is only the productivity of capital that evolves in most NW models. Instead the costs of variable inputs per unit of capital are fixed at

$$c = \frac{A^{\mathrm{K}}}{A^{\mathrm{L}}}w + \sum_{i=1}^{m} \frac{A^{\mathrm{K}}}{A^{\mathrm{X}}_i} p_i. \tag{2}$$

The NW model's handling of the complexities of equation 1 by means of equation 2 is quite acceptable for the first generation of partial models of Schumpeterian dynamics. But when it comes to growth models and the second generation of models of Schumpeterian dynamics, the truncated treatment of evolving Leontief technology seems less adequate. In both cases we need an answer to what may be called *the paradox of Leontief technology*: complex economies function smoothly although they make pervasive use of Leontief technology and although it seems practically impossible to avoid knife-edge problems in such economies. A core task of a generalised NW model should be to solve this paradox.

3 The pure-labour version of the Nelson–Winter model (LNW)

3.1 The need for simplification

The presentation of evolving Leontief technology has prepared the ground for the multiactivity generalisation of the NW model but it has also hinted at complexities that may blur things up at a too early stage of analysis. This will get much worse if we proceed immediately to the generalisation. The reason is that even what Malerba et al. (1999, 4) call "the first generation of mostly very general evolutionary economic models" have aspects of specialisation that make them difficult to handle and even more difficult to generalise from. In this situation there is obviously a need of applying the basic KISS principle from modelling folklore: "Keep It Simple, Stupid" (Axelrod 1997, Varian 1997). In accordance with this principle the generalisation will take place by means of successive approximations. In rough outline the modelling strategy can be depicted as a two-step procedure: first a simplification of the NW model in the present section and then the multiactivity generalisation in section 4.

The major victim of the simplification will be capital that functions as an extra constraint in the Leontief production function. It seems appropriate to begin by focussing on knowledge as the central constraint in NW models by simply leaving out capital. The resultant model may be called a labour version of the NW model or, for short, the LNW model. Since most researchers have apparently considered the treatment of capital a cornerstone of the NW model, the LNW model represents quite a radical approach.[13] For such researchers it should be emphasised that we can quickly reintroduce capital into the LNW model if and when it is found to be essential. A further consolation is that the LNW model still has the stock of knowledge left to give the firm its unique identity.

One of the advantages of initially leaving out capital is that we at the same time remove the special NW model features that are intended to ensure that monopoly does not emerge (see endnote 10). The avoidance of monopoly through restraint on capital accumulation is heavily influencing the structure of the NW model and this makes it difficult to study the pure logic of evolutionary processes. For that reason Nelson and Winter study these more general processes through a special family of formal NW models (see box 1). The initial removal of capital from the standard NW model is an alternative strategy that provides an LNW model that allows us to study these processes both analytically and through computer simulation. An important aspect of this study it the tendency toward monopoly both in a non-innovative economy (with initial productivity differences) and under different innovation–imitation regimes.

The development of the LNW model is not a goal in itself but rather a roundabout way of preparing section 4's development of the multiactivity generalisation of the NW model. But this roundaboutness has several advantages. First, the LNW model provides a bridge between the NW model and the multiactivity NW model. Second, the LNW model seems to be useful as a pedagogical device, both in itself and when preparing for studying the multiactivity NW model. Third, the LNW model is an important building block in the development of the generalised model multiactivity NW model that, furthermore, can easily be reduced to the LNW model. Fourth, the LNW model is an important starting point for the incremental implementation for computer simulations of the complex generalised model.

3.2 The specification of the LNW model

The basic assumption underlying both the LNW model and the full NW model is that because firms have limited information and are boundedly rational, they have to follow routines in their productive activities and in their market-related decision making. The starting point is to specify the (effects of the) routines that are related to the transformation of factor inputs (at the moment only labour) to final output. Her we in both models distinguish between evolving routines and fixed routines. The evolving routines are represented by real-valued productivities—A^K_{jt} and A^L_{jt} in the NW model, but only A^L_{jt}, or simpler A_{jt}, in the LNW model. The fixed routines are represented by a number of fixed parameters. Among these are the research intensity rule of thumb r_j, which relates research efforts to the level of capacity, and related rules of how to divide research into subactivities (e.g. innovative R&D and imitative R&D). If the overall model provides a possibility for the long-term evolution of the individual firms, then even these rules may be transformed into evolving variables (see e.g. the evolution of r_{jt} in Winter 1984 and in Silverberg and Verspagen 1994).[14]. In the following we shall concentrate on the evolving production routines that have the character of what some biologists call replicators (cf. Dawkins 1989). This name is appropriate because it suggests that the production routines or techniques can be replicated by new employees in the firm (after they have expecienced the necessary disclosure and learning). This replicator argument points to constant returns to the

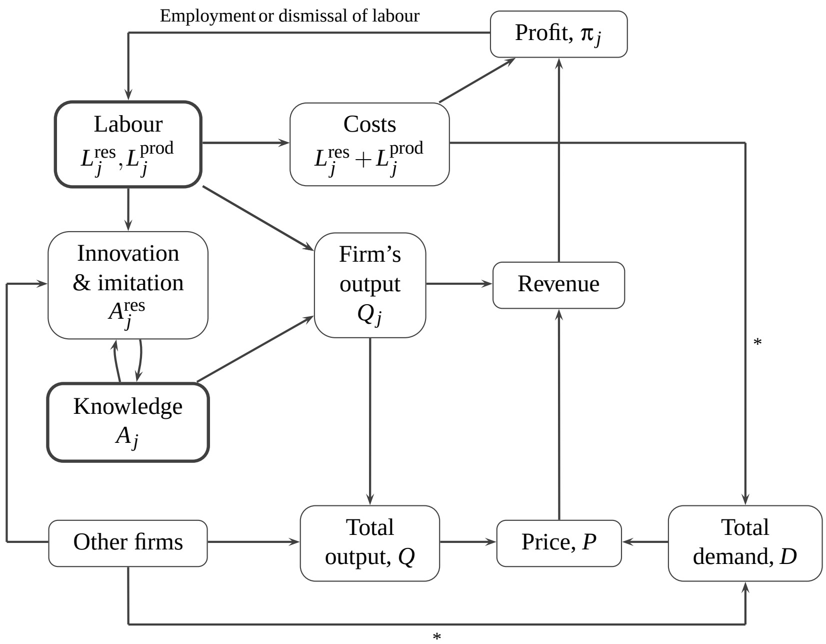

Fig. 2 Structure diagram of the pure labour version of the Nelson–Winter model of growth. The diagram puts an emphasis on a particular firm j, while aggregates are placed in the last row. An arrow from x to y should be read 'x codetermines y'. In the case of costs, the codetermination is simple: since the wage rate $w \equiv 1$, costs are simply equal to the quantity of labour. If the two arrows marked by an $*$ are removed, we have an LNW model of isolated industrial dynamics that can be compared with figure 1 on page 6.

scale production. The replicator argument also suggests the core of a theory of the firm that is basic for evolutionary modelling. According to this theory the firm is a repository of routines that it as far as possible tries to improve and keep for itself. The aggregate result of this firm-level behaviour with respect to knowledge about routines is that firms show a good deal of heterogeneity of behaviour.

Where the LNW model differs from the NW model is with respect to which production factors that—apart from knowledge—are included. In the standard NW model physical capital is the production factor that is fixed in the short run and thus at the centre of the firm's decision making while labour and intermediate goods are variable factors that can be treated quite summarically (see equation 2). In the LNW model all production factors are left out except knowledge and labour. In order to stick to the basic structure of the NW model, this means that the labour L_{jt} takes the place of capital as a state variable that is fixed in the short run. This means that the LNW model includes long-term employment contracts that can only be terminated at the end of each period. Real-life employment contracts are of course much more diversified, but the tendency that firms treat labour as a state variable is quite wide spread. There are also other reasons for firms to keep their employment as fixed as possible. Among these reasons are that there are skilling costs in order to qualify newly hired labour for the routines of the firm, that dismissed employees bring their skills to other firms, and that labour may not be available for suddenly needed expansion. The present version of the LNW model does not implement these underlying mechanisms behind labour as a state variable (see e.g. Ballot and Taymaz 1997), but they give a motivation for the emphasis on labour.

Another consequence of the concentration of labour and knowledge is that we can specify the emergence of new firms in a very simple way. In the NW model a supplement to investment restraint as a method of controlling concentration is to add a queue of potential entrants that actually enter the industry according to more or less plausible rules (cf. e.g. Winter 1984, 283–288). In the LNW model the incumbent firms of the economy/industry form an endogeneous source for the creation of new firms. Such new firms are simply formed by fissions of old firms because of intraorganisational conflict.

The functioning of the LNW model is depicted by figure 2. In this figure the last row of boxes deals with the economy or the industry as a whole while the rest of the figure concentrates on a particular firm j. In the following we start by considering (I) the markets for labour and output. Then we turn our attention to a particular firm j that is basically characterised by two state variables so that in every period t its actions start from its labour L_{jt} and its knowledge A_{jt}. On this basis the firm applies its set of decision rules to perform three sets of tasks that are shown in figure 2: (II) production and calculation of profit, (III) R&D outcome, and (IV) employment or dismissal of labour as well as fissions of firms. To analyse the dynamics of the model we also need (V) some statistics. These aspects of the LNW model will be dealt with in turn.

LNW-I. The markets:

1. The *labour market*:
   a. All of a firm's employees L_{jt} can perform both production tasks and research and they receive the same wage rate.[15]
   b. Employment contracts are based on a fixed wage rate that is paid at the end of the period (i.e. in arrears). Labour is the *numéraire* of the system, i.e.

$$w \equiv 1. \tag{3}$$

   c. Employment contracts can be terminated by the firm at the end of each period. It is assumed that dismissed labour can immediately be transferred to firms that want new employment.
   d. The firm ensures its ability to pay by adjusting its employment so that all its present revenue will be used for payments in the next period (see equations 9 and 13). The model is constructed so that the net aggregate change in employment is zero.[16] This functioning of the model means that a fixed aggregate quantity of labour is always employed,

$$\sum_{j=1}^{n_t} L_{jt} = L. \tag{4}$$

2. The *output market*:
   a. The *supply of final output* is the sum of the output of the n_t firms,

$$Q_t = \sum_{j=1}^{n_t} Q_{jt}. \tag{5}$$

   b. The *demand for final output* is the sum of all incomes obtained by labour, i.e.

$$D_t = \sum_{j=1}^{n_t} wL_{jt} = L. \tag{6}$$

   c. The *price final output* is the market clearing price. Since the monetary income is fixed, we have unitary elasticity of demand like in the NW model. In the LNW model this is specified as

$$P_t = D_t/Q_t = L/Q_t. \tag{7}$$

LNW-II. The firm's production and profit:

1. *Division of labour*: The firm divides its labour L_{jt} into two activities, production and research, according to a fixed decision parameter r_j.[17] Labour for production is $L_{jt}^{\text{prod}} = (1-r_j)L_{jt}$ and labour for research is $L_{jt}^{\text{res}} = r_j L_{jt}$.
2. *Production function*: The firm's production workers, L_{jt}^{prod}, produce output according to the firm's labour productivity and a full-capacity utilisation rule, i.e.

$$Q_{jt} = A_{jt}L_{jt}^{\text{prod}} = A_{jt}(1-r_j)L_{jt}. \tag{8}$$

3. *Production costs*: Since $w = 1$, the firm's total costs are simply $L_{jt} = L^{\text{prod}}_{jt} + L^{\text{res}}_{jt}$.
4. *Profit*: The firm sells all its output at the market price P_t. Thus it obtains the profit

$$\pi_{jt} = P_t Q_{jt} - L_{jt} = ((1 - r_j) A_{jt} P_t - 1) L_{jt}. \tag{9}$$

LNW-III. The firm's R&D outcome is modelled as a two-stage stochastic process:

1. *Probability of a research success*: The success or failure aspect of R&D is modelled as a stochastic variable $Z_{jt} \in \{0, 1\}$, where $Z_{jt} = 1$ means success and $Z_{jt} = 0$ means failure. The firm's research workers L^{res}_{jt} have a fixed productivity that is measured as the average number of successes per period per researcher, $1/\lambda$. The result of the firm's total research activities is modelled as a Poisson process with average waiting time for a success equal to λ times the number of researchers.[18] Thus

$$\text{Prob}(Z_{jt}{=}1) = \lambda r_j L_{jt}. \tag{10}$$

2. *Methods of research*: The research workers apply different R&D methods according to fixed parameters that determine the degree to which the researchers focus on different ways of improving knowledge: (a) cumulation of the firm's own knowledge, (b) imitation of the leading firm in the industry, (c) application of the industry's average knowledge, and (d) application of general knowledge (see section 3.4).
3. *Outcomes of research*: Firm j's fixed degree of emphasis on method x determines directly the probability that an R&D success is obtained by method x. The core method in the present paper is cumulative knowledge. In this case the outcome of a success is basically drawn from a normal distribution with mean determined by the firm's present productivity A_{jt} and standard deviation as a constant σ. To ensure scale-independent research outcomes, we in the normal distribution set the mean to $\ln(A_{jt})$ and then use the inverse exponential function to find the result. Thus we have

$$A^{\text{res}}_{jt} = \begin{cases} 0, & \text{if } Z_{jt} = 0 \\ e^{\text{normal}(\ln(A_{jt}),\sigma)}, & \text{if } Z_{jt} = 1. \end{cases} \tag{11}$$

4. *Productivity change*: The firm's productivity in the next period is the maximum of the existing productivity and the potential productivity obtained by R&D, i.e.

$$A_{j,t+1} = \max(A_{jt}, A^{\text{res}}_{jt}). \tag{12}$$

LNW-IV. The firm's labour accumulation and fissions:

1. *Labour accumulation*:
   a. *Mobility*: The LNW model has no labour mobility that is motivated by the employees (see the above specification I.1.c). This assumption means that employment contracts will continue indefinitely unless the firm makes dismissals. On the other hand, labour has full mobility from dismissing firms to employing firms.
   b. The firm's *accumulation of labour* is based on the rule that it spends all its renevue for the next period's payment of labour. This means that a firm that has $\pi_{jt} = 0$ will have an unchanged stock of labour, a firm with $\pi_{jt} > 0$ will make new employment contracts, while a firm with $\pi_{jt} < 0$ will make dismissals. The LNW model assumes that this process takes the simple form

$$L_{j,t+1} = (1 + \pi_{jt}) L_{jt}. \tag{13}$$

2. *Fissions*:
   a. Any firm may split into two firms that both inherits the level of productivity of the mother firm.
   b. The probability of a fission is ψ_t. In much of the discussion we may think of $\psi_t = 0$ for all periods. In other cases it is useful to let ψ_t reflect a theory of intrafirm conflict, so that it increases with a measure of dissatisfaction formulated like e.g. in Winter (1984, 281 f.). In still other cases we just want an ad hoc mechanism to avoid monopoly, so we

let the probability of fission be determined by a concentration index (cf. LNW-V.3), e.g. $\psi_t = f(H_t^{\text{inv}})$.

c. The split of a mother firms labour force is made in accordance to the uniform probability density function for the interval [0,1]. The firm with the largest employment inherits the identification number of the mother firm, while the smallest of the new firms gets the identification number $n_t + 1$.

LNW-V. Main statistics:

1. *Market shares*:

$$s_{jt} = Q_{jt}/Q_t.$$

2. *Employment shares*:

$$S_{jt} = L_{jt}/L.$$

3. *Concentration index*: The market concentration is measured by the inverse Herfindahl index for employment shares H_t^{inv}. This is the number of equal-sized firms that would have had the same Herfindahl concentration index as was found for the actual firms,

$$H_t^{\text{inv}} = \frac{1}{\sum_{j=1}^{n_t}(s_{jt})^2}.$$

4. *Mean productivity*: The capacity-weighted mean of the productivities is

$$\bar{A}_{s_t} = \sum_{j=1}^{n_t} s_{jt} A_{jt}.$$

5. *Variance of productivities*: The capacity-weighted variance of the productivities is

$$\text{Var}_{s_t}(A_t) = \sum_{j=1}^{n_t} s_{jt}(A_{jt} - \bar{A}_{s_t})^2.$$

6. *Static inefficiency index*: This index is defined as one minus the ratio between the actual aggregate output and the output obtainable through best-practice technology, i.e.

$$Q_t^{\text{gap}} = 1 - \frac{Q_t}{A_t^{\max} L}.$$

The main difference between the above specification of the LNW model and the standard NW model (cf. figure 1 on page 6) is obviously that we have left out capital and that labour has taken its place as a state variable. This change is coupled with a radical simplification of the investment function. In the NW model an investment function with monopolistic restraint is determining change in capacity (see footnote 10). In the LNW model the simplest possible investment function is applied (see equation 13). This function simply says that of the firm has had negative profits in period t, it makes an adjustment of its labour force so that it is able to pay its employees in period $t+1$ (out of the revenue obtained in period t). Similarly, a net gaining firm will expand its employment in order to spend all profits (and the rest of its revenue) on labour payments. Since aggregate profits are zero, no change in the aggregate employment will take place. Thus the LNW model focuses strictly on the change of the firms' employment shares and on the single factor that determines these shares: the relative productivities. This concentration makes it easy to add another important feature of the LNW model. In the NW model new firms are added through a queue of outsiders, but in the LNW model new firms are formed by fissions of old firms and simply inherits the productivities of the mother firms.

The movement of the productivities in the LNW model is defined by equation 12 and the related equations. These equations are (except for a few simplifications) very close to those of the NW model. But because of the simplified investment function, it it easier to study their effects. In the simplest case, where we have no research results (since $r_j = 0$ for all firms), productivity will stay unchanged indefinitely. If all firms initially have the same productivity level (and we thus have a static inefficiency of zero), no profits will be made and the initial employment shares will remain unchanged. If, on the other hand, firms have initial productivity differentials, the best performer k will ultimately obtain a monopoly position ($s_{kt} \to 1$ for $t \to \infty$). We shall consider this case in section 3.3. After these preliminary cases we shall in section 3.4 turn to the core issues: how the evolution of the productivities is determined by the different R&D methods and how this evolution in turn determines the evolution of the employment shares. Here we shall see that because increasing returns to the application of R&D results is implicit in the model's Leontief production function, monopoly is the normal outcome of the evolutionary process.

3.3 Fisher's theorem and replicator dynamics in the LNW model

Before we turn to the topic of the R&D based evolution of the Leontief technology in the simplified LNW economy, it is useful to consider what happens if there are fixed but different productivities in a system of firms that compete in the market for a homogeneous good (with no niches to hide in). In this case the LNW model becomes deterministic, as we would like for a first exploration of a model (cf. Sterman 2000). In this context we shall, as it has been suggested above, demonstrate that the result of the competition between firms (that show no restraint on their investment as market shares grow) is that the firm with the highest productivity will in the end become the only producer. This result is reminiscent of an important result in evolutionary biology (Gausse's principle): that in one selection niche there is only room for one species. We shall also see that this result is the outcome of a general method of evolutionary analysis (cf. Sober 1984), which was originally developed in R. A. Fisher's works and which was adapted for economic evolution by Nelson and Winter, especially in their simple models of economic selection for a two-technology case and a multitechnology case (in box 1 on page 5 these models are named as NW10.1 and NW10.2).

The core result of the evolutionary analysis is Fisher's (1999[1930], 35, emphasis removed) "fundamental theorem of Natural Selection . . . : The rate of increase in fitness of any organism [or, rather, species] at any time is equal to its genetic variance in the fitness at that time." This theorem was put forward by Fisher while he was developing the modern synthesis between Mendelian genetics and Darwinian evolutionary theory as well as related core methods of modern statistical analysis. The Fisher theorem and the related Fisher principle are in no way bound to genetic selection (cf. Metcalfe 1994). Instead the theorem and the principle reflect a general method of evolutionary analysis that has its focal point in the dynamics of statistical characteristics (like mean and variance) of the replicated characteristics present in populations of competing organisms or firms. Because of the specificity of the NW model, Nelson and Winter (1982, 243) had to develop auxiliary formal models to obtain similar results. But the LNW model allows us to derive the results without a change in model specifications.

Let us in the framework of the LNW model think of a fixed number of n firms that in period $t = 1$ all have the same employment ($L_{j1} = L/n$). The firms have different productivities and there is no R&D ($r_j = 0$ for all firms), so their productivities are fixed. The firms are, furthermore, ordered so that the first firm has the highest productivity, i.e. $A_1 > \ldots > A_j \ldots > A_n$. Since the firms operate in a context of a fixed input price, they also have fixed unit costs, e.g. $c_j = w/A_j = 1/A_j$. Thus the ordering of the costs is $c_1 < \ldots < c_j < \ldots < c_n$.

In the first period all firms have the same employment share, $S_{j1} = 1/n$, and the same costs, L_{j1}. But the firms differ with respect to their market shares, s_{j1}, and profits, π_{j1}. The problem of different profits will continue in the subsequent periods, for although the price will fall as output increases, the cost differentials stay unchanged.[19] This means that in all periods firms will have different growth rates. It is convenient to perform the analysis in terms of output change, so we start from profits per unit of output, $\pi_{jt}/Q_{jt} = P_t - c_j$. Firms want to expand their capacity if they have a positive profit ($P_t > c_j$), while firms contract if they have a negative profit ($P_t < c_j$).

To analyse the evolutionary process of the fixed-productivity LNW model we need to change our perspective from the destiny of individual firms to the change in the probabilistic properties of the population of firms and its output (cf. part V of the above specification of the LNW model). To make it easy to handle the population, let the output be a discrete good. Then we can in period t from the aggregate output take a random instance of the good and ask what is the probability that this sample is produced by means of productivity A_j and thus by the unit cost c_j. This question puts all firms with unit cost c_j into an equivalence class (cf. Winter 1987), but in the fixed-cost version of the LNW model only one firm j has unit cost c_j. Thus the probability that the random instance of the good is produced with unit cost c_j is equal to firm j's market share $s_{jt} = Q_{jt}/\sum Q_t$. If we define X_t as the random variable for which $X_t(c_j)$ is the probability that an individual sample of the good is produced by means of unit cost c_j, we may also say that

$$\mathrm{Prob}(X_t = c_j) = s_{jt}.$$

The next question is what we should expect to be the unit costs of an individual sample of the good. The answer is obviously the mean value of the unit costs in period t, or formally

$$\mathrm{E}(X_t) = \bar{c}_{s_t} = \sum s_{jt} c_j.$$

In evolutionary analysis we also need to study the higher moments of our random variable. Presently we shall stick to the variance of the unit costs for which it is convenient to give both the definition and an alternative expression

$$\mathrm{Var}(X_t) = \sum s_{jt}(c_j - \bar{c}_{s_t}) = \sum s_{jt} c_j^2 - \bar{c}_{s_t}^2. \tag{14}$$

After these preliminaries we come to a core question of evolutionary analyses: what happens over time to the expected value of the LNW random variable X_t? To answer this question we have to perform several steps. We start by analysing firm j's growth rate

$$g_{s_{jt}} = \pi_{jt}/Q_{jt} = (P_t Q_{jt} - L_{jt})/Q_{jt} = (P_t A_{jt} L_{jt} - L_{jt})/(A_{jt} L_{jt}) = P_t - 1/A_{jt} = P_t - c_j,$$

i.e. the growth rate of a firm is simply the distance between the market price and its unit costs. Given this result we look for the mean growth rate of the whole population of firms

$$\bar{g}_{s_t} = \sum s_{jt} g_{s_{jt}} = \sum s_{jt}(P_t - c_j) = P_t \sum s_{jt} - \sum s_{jt} c_j = P_t - \bar{c}_{s_t},$$

since $\sum s_{jt} \equiv 1$. By combining the last two results we find the growth rate of firm j's market share

$$\dot{s}_{jt} = s_{jt}(g_{s_{jt}} - \bar{g}_{s_t}) = s_{jt}(P_t - c_j - P_t + \bar{c}_{s_t}) = s_{jt}(\bar{c}_{s_t} - c_j), \tag{15}$$

which is simply the *replicator equation* with unit costs interpreted as fitness indexes. [20] Such a replicator equation embodies the distance-from-mean principle of evolutionary analysis. Qualitatively this means that that in any period we have two types of firm, the growing firms with undernormal unit costs and the shrinking firms with overnormal unit costs.

The replicator equation 15 has many nice properties (cf. Silverberg 1988, Metcalfe 1994, and Hofbauer and Sigmund 1998, Ch. 7). One of them is that Fisher's theorem holds so that the mean fitness of the population increases along any trajectory of this equation. More specifically, we find that the growth rate of the capacity-weighted mean of fitnesses of such a trajectory can be expressed in terms of the capacity-weighted variance of the unit costs—if we remember the variant expression for the variance from equation 14, the result of equation 15, as well as the definition $\sum s_{jt} \equiv 1$. To obtain the result we need the following substitutions into the definition of the growth rate of the mean of the unit costs:

$$\dot{\bar{c}}_{s_t} = \sum \dot{s}_{jt} c_j = \sum s_{jt}(\bar{c}_{s_t} - c_j) c_j = \sum s_{jt} c_{jt} \bar{c}_{s_t} - \sum s_{jt} c_{jt}^2 = \bar{c}_{s_t}^2 - \sum s_{jt} c_{jt}^2 = -\mathrm{Var}(X_t). \tag{16}$$

The result of equation 16 is the LNW model's version of Fisher's "fundamental theorem of natural selection" that the rate of reduction of the mean unit costs is simply equal to the variance of the population's unit costs. If the capacity-weighted variance is high, mean costs fall quickly. As a result variance decreases and mean costs fall more slowly. When variance reaches zero, there is no further change in mean costs and the lowest-cost firm has obtained monopoly.

Fisher's theorem reflects, as already mentioned, a basic method of evolutionary analysis. The idea is to consider a core characteristic of the population as a random variable and then to study the dynamics of this random variable. Since economic phenomena are not as neatly organised as

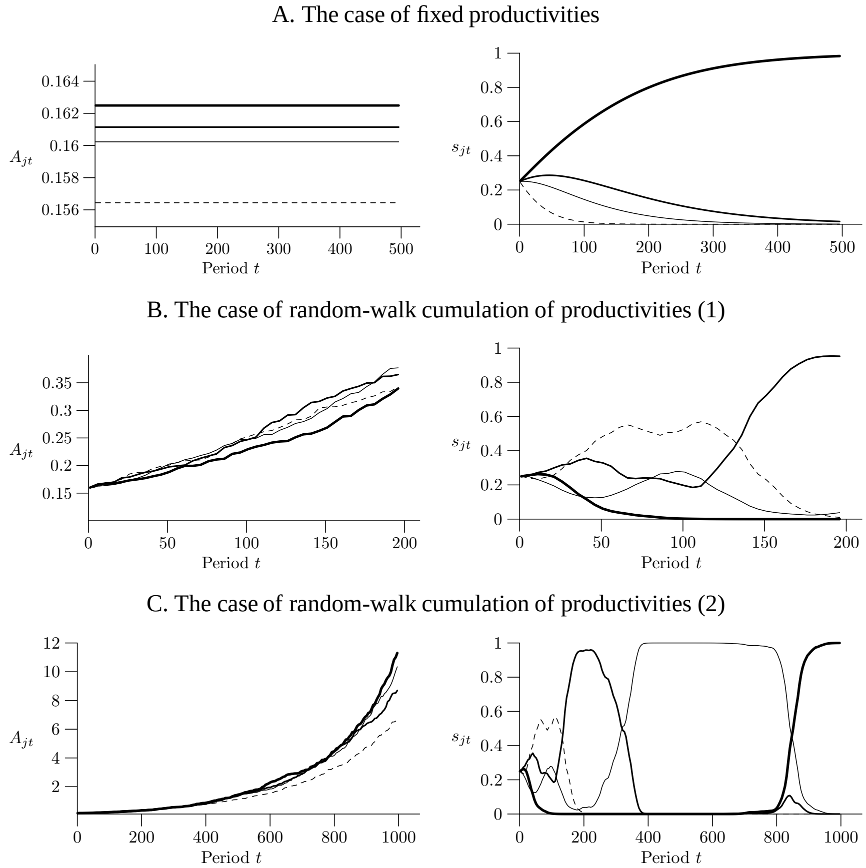

Fig. 3 Simple dynamical patterns in the LNW model: (A) Pure replicator dynamics. (B) Random-walk improvements of productivities that apparently lead to monopoly. (C) Continuation of the random-walk simulation that demonstrates that B was not really a lock-in situation.

biological ones with respect to evolutionary studies, we might easily be led to argue as if the concept of fitness is a mere tautology, since our formulations sometimes sound as if we are simply talking about "the survival of the survivor" (Popper 1972). As pointed out by e.g. Winter (1987) this will not be the case if we think clearly. Our first task is to group our data into a certain kind of equivalence classes defined in terms of both copying and selection. Then we calculate the process of change of each equivalence class in terms of relative frequencies—considered as being the revealed fitness of this equivalence class. Finally, we look for an explanation of the revealed fitness; this explanation we call the theoretical fitness.

3.4 The influence of innovation and imitation on the dynamics of the LNW model

A basic conclusion from the study of replicator dynamics is that the selection process uses up its own fuel. So if we observe a case of continuing evolutionary change, we infer that the system includes a mechanism that generates new variety in pace with the variety reduction due to the selection mechanism. In evolutionary economic models it is customary to identify this variety creating mechanism with R&D, but this is mainly for convenience since formal R&D is just one of several contributors to variety creation. Therefore, it is important to note that both the standard NW model and the LNW model are designed to function as testbeds for different 'regimes' of variety creation (cf. the LNW specification's part II on page 13).

The starting point for our short discussion will be a simulation of the LNW replicator dynamics for which we have already found analytical solutions. Although our understanding thus does not really need computer simulation experiments, it is helpful to use such simulations as benchmarks for the exploration of the more complex models. An example of a fixed-productivity simulation of the LNW model is shown in part A of figure 3. In the left panel we see the fixed productivities of 4 firms and in the right panel we see the trajectories of the market shares. From panel A.1 it can be seen that the random number generator has been used to create the initial productivities of the four firms. These small productivity differences may appear to be of little importance, but their cumulative effects can be followed in panel A.2. Here we see that the firm with the highest productivity (indicated by a thick line) in the end of the simulation is pretty close to a market share of 1. On the other hand, the lowest-productivity firm (with the dashed line) quickly disappears. The two in-between firms are more telling for the replicator dynamics. In the beginning they show little change in market share because that have close to average productivity; one of them actually increases its share for a while. However, as the strong firm grows and the weak firm shrinks the share-weighted mean of the productiveties increases and therefore the position of the in-between firms is weakened. Further simulation experiments can easily be performed in any programming system.

In panel B and C of figure 3 we turn to the case of a truncated random walk in the productivities. This means that each firm will in each period have a small but random upward move in its productivity, no matter whether the firm has a large or a very small market share. This means, as we see en panel B.1, that all productivities moves upward in a way so that the distance between the firms would show a random walk. In panel B.2 we see the turbulence of market share, but also that one firm appears to take over in the end. But as the random walk was defined there will sooner or later some a sequence of random numbers for another firm that will bring it up in front. In panel C.2 we actually see a succession of near-monopoly positions in the industry. This warns us against telling “just-so stories” about phenomena that are fully of a stochastic nature and where firms probabilistically are exactly alike. Thus the results are like in Arthur’s (1989) random walk case.

The next issue is whether we can find an equivalent to Arthur’s case of lock in because of increasing returns. The production function of equation 8 shows constant returns to scale and the research activity of equation 10 has also constant returns to scale. If we, however, combine equation 8 and equation 12, we see the source of increasing returns in the LNW model (and the NW model): the model is characterised by increasing returns to the application of an innovation, which is produced under fixed (probabilistic) costs. No matter whether a given increase in productivity is applied in a large or a small firm, its effect covers the whole of the productive capacity in the next period. Thus the most effective use of an innovation is to apply it in a firm with a market share of 1. This is the logic underlying the simulations depicted by figure 4.

In panel D.1 of figure 4 we see the result of a simulation of the usual 4 firms where there is a gradual removal of firms from the progress of productivities to the fixed-productivity state that we saw in the treatment of replicator dynamics. There are two interrelated reasons for this result. First, firms uses a fixed share of their labour force for research, so a large firm will spend more than a small firm and this have a larger probability of success (contrary to the random walk case). Second, as mentioned above the large firm applies its research results more efficiently than a smaller firm. As time progresses weak firms become smaller and smaller and their probability of innovative success moves toward zero. Panel D.2 demonstrates that in the beginning of this process it is impossible to predict who is going to become the monopolist. Thus we see that the firm marked with the dashed line has a period of market share leadership before the firm with the thick line takes over.

The two last simulations represent attempts to avoid the march toward monopoly. In the case of panel E we have made a distinction between two firms that have no R&D expenses while the two others perform research. Initially the firms without R&D produce more, obtain a higher profit, and grows at the expense of the firms with research. One of the R&D firms is lucky to obtain an innovation while the other R&D firm disappears more quickly than the firms without research. It is not difficult to make simulation setups where all R&D firms disappears, but then empirical fact of evolution also disappears from the simulation.

The last simulation (panel F) represents yet another way of weakening the dominant firm, namely by making it very easy to imitate the position of the leading firm. This means that all firms follow a narrow band of productivity growth, until they drop out—one after one. The reason is that both

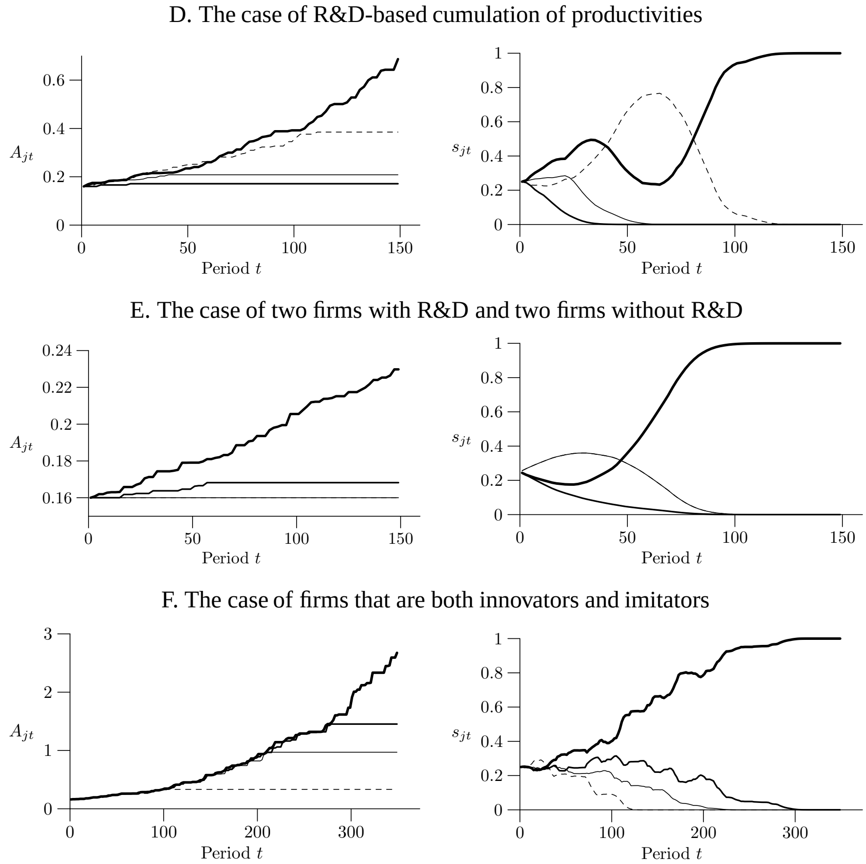

Fig. 4 Dynamical patterns in the LNW model due to R&D based innovation and imitation: (D) R&D results have their mean value in the present productivity of the firm. (E) Two firms perform R&D like in panels A, while the two other firms have no R&D and thus obtains an initialy higher profitability. (F) All firms perform innovation and imitation. Imitators can in one step reach the productivity level of the leading firm.

innovation and imitation requires resources to perform imitations, so in the end we still see that one firm takes over. So even if imitating firms obtain the productivity of the leading firm, they do not become better by imitation alone. Instead they succumb after periods of 'bad luck'.

3.5 The limits of the one-sector LNW model

Although the LNW model has a few complexities, and interesting features (like the fissions of firms), it is by no means an answer to the present papers call for a generalising strategy as a complement to the "history-friendly" modelling strategy. Instead it is an important stepping stone between the NW model and the multiactivity version of the NW model. It also has some advantages for academic teaching. Here it is obvious that the richness of NW models is to some extent a hindrance for understanding, even for advanced students. The reorganisation and simplification of the NW model by means of the LNW model might help to change this situation. It might even give a small push toward the production of the seriously needed textbook exposition of the Nelson–Winter tradition and toward a set of simulation exercises for specially interested students.

The main issue is, however, how to proceed toward a multiactivity NW model, the MNW model. Here is is important to note that given the LNW model, the major step in the construction of the MNW model is not difficult, to split up the aggregate production activity and research activity of the LNW model into a number of activities that are characterised by their own productivities and by related R&D activities. When this step has been performed, we can turn to many of the other missing issues in the LNW model. One issue is to bring more structure into the LNW model so that the individual activities form some sort of input–output pattern. A related issue is to determine the possibilities of an endogeneous emergence of this structure, just as Winter (1984) follows the life cycle of an industry. Furthermore, there is the issue of providing more interesting markets than are found in the LNW model and, in that connection, endogenously emerging markets. Finally there is the issue of providing more interesting R&D strategies and of exploring the related coordination problems.

Before we leave the LNW model it should be pointed out that its possibilities are by far exploited by the present paper. Since it is largely its specification of capital accumulation that confines the application of the NW model to industrially oriented economic dynamics, the LNW model might help to turn to other applications. For instance, it is partly the capital part of the interesting model by Silverberg and Verspagen (1994) that limits its interpretation. The broadening of the interpretation is actually implicitly suggested by e.g. the choice of 8,000 periods for the simulation runs, so that the reader is pointed towards basic economic history and the very long-term emergence of and intensification of R&D activities. Still their paper's concept of physical capital points to a modern setting. Since the LNW model does not presuppose capital of the modern type, it can more easily by adapted for a large area of potential applications. To take a radical example (see box 2): the model can be used for a crude handling of "stone age economics" (cf. Sahlins 1974, Boyd and Richerson 1985). As suggested at the end of the box, even the stone age example points towards a multisectoral development of the simplistic LNW model.

4 The multiactivity version of the Nelson–Winter model (MNW)

4.1 Intrafirm diversity as a source of interfirm diversity

There seems to be two major strategies in the search for a solution to the diversity paradox and the Leontief technology paradox of the NW model and many other evolutionary models. The first strategy is to turn directly to the diversity of the market environment. The second strategy is to start from the inner diversity of the firms. The first strategy finds rather little support in most of Nelson and Winter's writings; in Winter (1991) we even see an explicit contrasting of production-oriented evolutionary theorising and more or less non-evolutionary transactionist analyses. On the other hand, even the quickest inspection of Nelson and Winter's (1982) *Evolutionary Theory* demonstrates that the second strategy appears to be implicit in the book's whole approach and not least in Part II's account for the "Organization-Theoretic Foundations of Economic Evolutionary Theory". Here we e.g. read (p. 98) about the inner structure of a firm as so complex that it is helpful to start from an analogy to Schumpeter's "circular flow" within an individual organisation and then gradually turn to the processes of intrafirm change. We further see that "[p]revailing routines define a truce, and attempts to change routines often provoke a renewal of the conflict which is destructive to the participants and to the organization as a whole" (p. 134). Finally, we see an emphasis on the firm's heterogeneous skills and competencies, an issue that is e.g. developed by Teece, Rumelt, Dosi and Winter (1994).

Although the emphasis on the inner diversity and the resulting coordination problems of firms in the foundations part of the *Evolutionary Theory* is not followed up in the development of the NW model, it is not difficult to see how such an application can be made. By starting from multiactivity firms, the task is to explain why and how individual activities become outsourced and coordinated by more-or-less clear-cut market mechanisms. In principle this disintegration may start from totally autarkic producer–consumer firms like in Adam Smith's (1976[1776], Bk. I, Ch. 3) story of the Highlands of Scotland where "every farmer must be butcher, baker, and brewer for his own family", and where a more and more complex market system may emerge in a slow and gradual manner by mutual and multilateral specialisations. But this is probably too far from the present concerns of the Nelson–Winter tradition. Instead we shall turn to the traditions in industrial economics and growth theory that relate to the Smith-inspired ideas of Marshall (1961[1890]) and Young (1928). Here there is an intense interest in the close relationship between the internal economies of firms and the external economies that arises from interfirm specialisation with respect to production and

Box 2 "Stone age economics" in a LNW model.

Compared to the standard NW model the LNW model has a structure that makes it much less bound to the *problematique* of industrial economics. To emphasise this point this box gives some suggestions in relation to the highly controversial topic of "stone age economics" (Sahlins 1974) that is also touched upon in Nelson's (1995) survey (cf. endnote 2).

In the standard LNW model a fixed labour force produces an increasing amount of output and thus an increasing per capita consumption. In classical economics and in evolutionary anthropology it is, on the contrary, an increasing labour force that produces a roughly fixed amount of output per worker. In this setting output functions as an input to the reproduction of the labour force and thus to population dynamics. It is not difficult to change the specification of the LNW model to deal with this issue.[21] Thereby we change the perspective away from Sahlin's rosy picture of "stone age economics" with short working hours and a sustainable birth rate in hunter–gatherer societies.

The simplest way of designing an LNW model for the 'stone age' environment of cultural-evolutionary adaptation is to abolish the two markets of specification LNW-I on page 12. Instead we shall think of n culturally and sexually isolated populations. Through cultural cumulation each of these populations increases its ability to produce the minimum output needed per capita. In this setting we have a non-changing P, the amount of output needed to reproduce one unit of labour. The 'profit' π_{jt} of a population is then the surplus output compared to the output needed for the reproduction of the present population. If the profit is positive, the population increases; if it is negative, the population shrinks. Thus we can even in stone age dynamics apply the formulation $L_{j,t+1} = (1+\pi_{jt})L_{jt}$ (equation 13), although with a totally new interpretation. As in specification LNW-III on page 13 this cumulation of labour is totally determined by the cumulation of knowledge. Under stone age conditions a major problem is how to ensure the reproduction of a given level of knowledge, but this issue will not be dealt with here. The precise conditions for the creating of new knowledge are even more difficult to specify, so we shall presently just assume that they are like in the LNW model.

According to evolutionary ecology (see e.g. Kingsland 1985, Futuyma 1998, Ch. 4) there are two types of environment in which this population dynamics take place. These types are named in terms of the parameters of the logistic equation and the Lotka–Volterra equations, where r means the intrinsic rate of natural increase and K means the carrying capacity of the environment. For the study of human populations the simple r-selection, where the population with the largest intrinsic birth rate comes to dominate, is probably of little interest. Instead we shall consider two subtypes of K-selection. As the LNW model is formulated, it best represents a simple K-selection environment where each population has filled out its own territory or niche, so that it can only expand by increasing its productivity in exploiting its given environment. This means that all populations will grow in pace with their cumulation of knowledge and productivity. In the simple case where all populations have the same chance of improving their knowledge (equations 10 and 11) they grow at different speeds according to their different rules of allocation of labour for the production of knowledge. In the very long run the population that find the optimal solution to the trade-off between direct production and knowledge production will become dominant. If there is also competition for territories and niches, this tendency will be strengthened. The tendency toward large populations may, however, be checked by fissions (cf. LNW-IV.2).

Because of their continued growth the populations will sooner or later enter into a complex K-selection environment with inter-population competition as well as intra-population competition. Here the knowledge cumulation tends to become a matter of survival of each population and its subpopulations. However, there will also be increased possibilities of cultural imitation and sexual interaction between populations. Furthermore, the populations have the possibility of specialisation and exchange. Thus we might obtain a somewhat more benign picture of "stone age economics". This possibility is sketched out in box 3 on page 33 in relation to the multiactivity version of the LNW model.

knowledge creation. To obtain a quick and concrete picture of these relationships, it is helpful to quote Young's (1928) description of his favourite example: the disintegration of the printing trade.

> The successors of the early printers, it has often been observed, are not only the printers of today, with their own specialized establishments, but also the producers of wood pulp, of various kinds of paper, of inks and their different ingredients, of typemetal and of type, the group of industries concerned with the technical parts of the producing of illustrations and the manufacturers of specialized tools and machines for use in printing and in these various auxiliary industries. The list could be extended, both by enumerating other industries which

> are directly ancillary to the present printing trades and by going back to industries which, while supplying the industries which supply the printing trades, also supply other industries, concerned with preliminary stages in the making of final products other than printed books and newspapers.

This story is Young's answer to the monopoly paradox that arose from Marshall's allowance into his system of economies of scale and that was brought into focus by Sraffa (1926). There is no real paradox as long as we allow into our models the indefinite divisibility of production activities. This divisibility often makes a small well-focussed firm more productive than a large firm with a broad scope of activities. Although concentration is a real process, the trend is broken by the evolution of markets for more and more intermediate goods that slowly undermine many of the industrial giants. This idea did not take a lead in the "Years of High Theory" (Shackle 1967), where the interest instead turned toward monopolistic competition based on consumers' preferences for diversity, but it has remained an important source of inspiration for e.g. Kaldor (1981) and Romer (1986).

The question is, of course, what Young's (1928) idea has to bring evolutionary modelling in the Nelson–Winter tradition. The answers are straightforward. First, it points to a solution of the diversity paradox that is more in like with the endeavours of the tradition than an immediate application of the Chamberlin approach. Second, the introduction of a varying borderline between intrafirm and interfirm activities suggests an answer to the Leontief technology paradox: if a firm cannot get its necessary inputs from the market, it can produce them in-house (although at higher costs).[22] Third, the evolutionary formalisation of the idea opens up new areas of "history-friendly" modelling. Fourth, the exploitation of the idea leads to an emphasis on intraorganisational issues and thus to an exploration of many of the ideas that were formulated verbally in Nelson and Winter's *Evolutionary Theory*. Fifth and finally, the approach supports Nelson and Winter's (1982) critique of general equilibrium economics more strongly than does the NW model. Even in their model one might ask whether an equilibrium will be met "at the end of the road". In a Smithian context Richardson's (1975, 357) answer is "that the end of the road may never be reached. ... For just as one set of activities was separable into a number of components, so each of these in turn become the field for a further division of labour." The opening up of these possibilities are part of the evolutionary process itself: "the very process of adaption, by increasing productivity and therefore market size, ensures that the adaptation is no longer appropriate to the opportunities it has itself created." (Richardson 1975, 358)

4.2 The basic structure of the MNW model

Although there seems to be clear needs for a multiactivity generalisation of the NW model (a MNW model), it is be no means simple to design such a model. A major obstacle is the tendency of modelling to become too ambitious with respect to the handling of the many interdependencies between the different production activities and knowledge areas. Thus Teece et al. (1994) tries to use a multiactivity and multicompetence approach to understand "corporate coherence" while Leijonhufvud (1986) and Langlois and Robertson (1995) go further into the structure of production chains. But in an evolutionary model even the simplest attempts of handling emerging production chains and emerging knowledge chains tend to become too complex for most analytical purposes (cf. the attempts in Andersen 1996a, 1996c). To move forward it seems necessary to apply the KISS principle and start from a radical simplification of the input–output structure of production and knowledge creation. This is at least the strategy chosen in the present development of the MNW model, whose basic structure is shown in figure 5—seen from the viewpoint of an individual firm.

The starting point is the LNW model as it was developed in section 3. As we saw in the specification of the LNW model (LNW-II and LNW-III on page 12) each firm has only one production activity and one R&D activity (combining process innovation and process imitation). To obtain multiple activities we can simply think of these LNW activities as being simple aggregates of m subactivities. Thus we have m production activities and m related R&D activities. The R&D activities function as in the LNW case. The only difference is that an individual innovation concerns only one of the

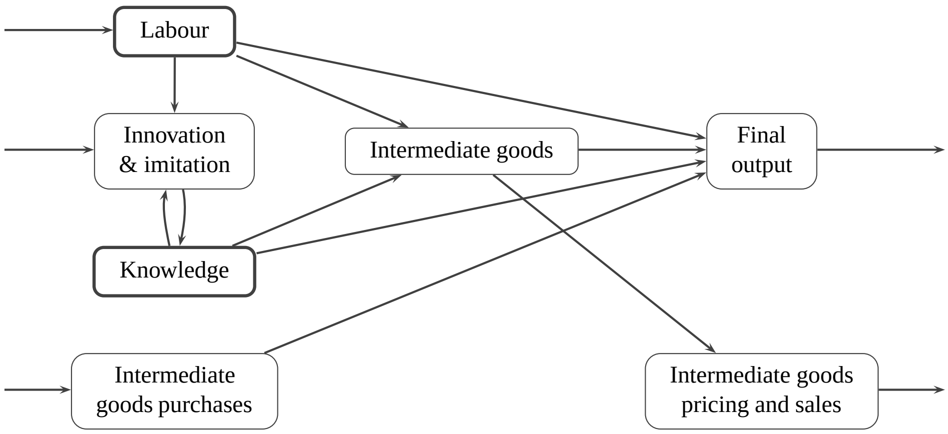

Fig. 5 Structure diagram that only covers a single firm's activities in the multisectoral Nelson–Winter model. An arrow from *x* to *y* should be read '*x* codetermines *y*'. The diagram may be compared with the firm-specific part of figure 2 on page 11.

activities, so the size of the productivity increase has to be m times as large to give the same overall productivity effect as in the LNW model.

The generalisation to m production activities is slightly more complex. The problem is how the different production activities should relate to the production of final output. The solution chosen in the MNW model is to have one production activity that combines $m-1$ intermediate goods into final output. This final good activity operates according to a Leontief production function that is very close to that of equation 1 on page 9. The only difference is that all intermediate goods are used up within a single period, so that there is no close equivalent to physical capital. The chosen Leontief function means that to produce one unit of final output, activity #1 needs one unit of each of the $m-1$ intermediate goods as well as $1/A_{1jt}$ units of direct labour. The production functions for the $m-1$ activities that produce intermediate goods are much simpler, since these activities use only labour and knowledge, so that $Q_{ijt} = A_{ijt}L_{ijt}$ (just as in equation 8 on page 12).

Compared to any input–output table as well as to the ambitions of e.g. Leijonhufvud (1986) and Langlois and Robertson (1995), the structure of the MNW model appears to be naively simplified. This is, indeed, the case if we only use the MNW model for the analysis of intrafirm issues. Let us, for instance, divide the overall production labour of an LNW firm into m equal-sized subactivities. To obtain the same results as in the LNW model, this decomposition presupposes that all subactivities have the same productivity, $A_{ijt} = mA_{jt}$. This can be seen from the following: In the LNW model we needed $1/A_{jt}$ units of labour to produce one unit of final output. In the MNW model we need $\sum_{i=1}^{m} 1/A_{ijt} = 1/A_{jt}$ units of labour, i.e. exactly the same. Concerning R&D things are equally simple. If the size of an innovation is m times that of the LNW model, the firm obtains the same aggregate productivity gain as before. So the MNW model may look as an unnecessary complication of the LNW model. There is, however, one crucial difference: Even though two MNW firms have exactly the same aggregate productivity, they need not and will not in practice be equal with respect to their productivity profile. The reason is, of course, that for stochastic reasons two firms will not have improved the same productivities to the same degree. So even two firms with the same overall productivities might gain from trade. Therefore, intermediate goods markets may emerge endogenously in the MNW model—simply because of the stochastic process of activity-specific productivity change.

The make-or-buy decisions and the sell-or-use decisions of firms in the MNW model are, in principle, quite simple. The potential seller of an intermediate good sets a supply price that covers its costs times a markup factor. The potential buyer compares the supply price with its reservation price (determined by its unit costs). If both parties gain from the exchange, a contract is made and the intermediate goods are supplied just in time for the finalisation of the final output in the period under consideration. This looks pretty straightforward, but from a modelling point things are more complex since we have to specify precisely how the system of intermediate good markets is functioning. It is, however, not difficult to specify an algorithm for the functioning of the intermediate markets. In the MNW model it is assumed that the intermediate market with the largest differences between supply prices and reservation prices comes first. Within each market it is assumed that the supplier with the lowest price comes first and serves as many as possible of the customers (from the end with the highest reservation prices) before the next cheapest supplier enters. In this way a precise market process takes place. To control the degree of trade in the MNW model, there is added an extra feature that is not normally dealt with in evolutionary models: transaction costs. These costs are modelled in the simplest possible way (cf. Yang and Ng 1993): if the supplier has costs that would give x_{ijt} units for in-house use, the purchasing firm only receives $X_{ijt} = (1-\kappa)x_{ijt}$ units of the good. If the transaction costs parameter κ is close to 1, it is practically impossible to obtain productivity differences large enough to motivate exchange. If κ is close to 0, even relatively modest productivity differences will lead to exchange.

The core issues of the MNW model are connected to R&D. As long as all firms are autarkic with respect to intermediate goods, the firm's choice of R&D specialisation is fairly easy (see section 4.5). But as soon as exchange emerges, the problem of R&D specialisation becomes pretty complex for the boundedly rational decision makers of the MNW model. The reason is that the firm cannot be sure whether it in the future will uphold its position with respect to sales and purchases of intermediate goods. Therefore, the question is whether the firm should strengthen its given positions by a narrow R&D specialisation or whether it is better to spread its researchers over a larger set of activities. In other words, the MNW model is a testbed for a large set of strategies of R&D specialisation. In close connecting to these R&D strategies are the pricing strategies for the suppliers of intermediate goods. The dynamic problem concerns the sharing of the ever-changing gains from exchange of intermediate goods. If the supplier gets a too large part of the gains, its customers will do relatively badly in the dynamical process of labour accumulation, and this will in turn influence the profits of suppliers with high markups. Another dynamical trade-off concerns the fact that a successful supplier may outgrow its chosen intermediate good market. In this case the successful firm has to take up other production activities, and this works best if R&D has prepared for the firm's path of expansion.

An additional difficulty for firms in the MNW model is that there is a possibility for a type of R&D whose results increases the number of intermediate goods. This so-called structural R&D results at first in an increase in the decomposition of the firm's production activities. This increased decomposition is made in a productivity-neutral way, so that it makes little sense for autarkic firms to engage in structural research. However, in an economy with exchange of intermediate goods, it may be very profitable to perform structural innovations since the first innovator will have a productivity advantage in that area (although there is a spillover to other firms so that they can easily reorganise their production). It is especially firms that have relatively strong positions in many knowledge areas that can easily benefit from decomposition since the initial productivity in the new area is influenced by the firm's general level of productivity.

4.3 The specification of the MNW model

In a certain sense the MNW model is a simple extension of the LNW model, which can be seen from the fact that the MNW model can easily be collapsed to the LNW model. This is an important property. First, it means that all the results obtained in section 3 hold for simplified versions of the MNW model. Second, since the characteristics of the LNW model have been developed in explicit relation to the NW model, we can also relate the MNW model to the NW model. Third, large parts of the specification of the MNW model have already been made in connection to the LNW model. This latter advantage will be exploited in the following presentation, where overlapping specifications shall not be repeated; instead the reader is referred to the relevant parts of the specification of the LNW model (starting on page 12). This shortness might be considered a disadvantage by those who just want to inspect the specification of the MNW model. However, the two specifications have the same design and there are a lot of cross references from the MNW specification to the unchanged parts of the LNW specification.

Before we turn to the MNW specification, it should be pointed out that it is not fully following the principle of simplicity. The reason is that one of the purposes of the specification is to give an impression of the potential richness of MNW models. To give an impression of the basic structure of the MNW model, the specification indicates which features that should initially be switched off or studied at extreme values. These remarks prepares the return in section 4.4 to the KISS principle (cf. page 10).

MNW-I. The markets:

1. The *labour market* is like in LNW-I.1 on page 12.
2. The *market for final output* is like in LNW-I.2 on page 12. The final good is #1, so the MNW version of the LNW price equation 7 is

$$P_{1t} = L/Q_{1t}. \tag{17}$$

3. The *markets for the $m_t - 1$ intermediate goods*:
   a. *Transaction costs*: All these markets are characterised by transaction costs according to Samuelson's "iceberg" transaction technology that means that if seller k supplies x_{ijkt} units of intermediate good i, the buyer j receives $X_{ijkt} = \kappa x_{ijkt}$ of the good, where $0 \leq \kappa \leq 1$. In simplified versions of the MNW model it is assumed that $\kappa = 1$ or $\kappa = 0$.
   b. *Demand for intermediate goods*: In period t the overall demand for intermediate good i is determined by the reservation prices of the firms and by their desired output of the final good $Q_{1jt}^{\text{desired}} = Q_{1j,t-1}$. Thus we have the stair-formed demand function

$$D_{it}(P_{it}) = \sum_{j=1}^{n_t} [\max(\frac{1}{A_{ijt}} - P_{it}, 0) Q_{1j,t-1}]. \tag{18}$$

   c. *Supply of intermediate goods*: All firms have a minimum price for the potential supply of intermediate goods. Firm k's supply price for good i reflects its unit costs $1/A_{ikt}$, the general transaction costs κ, and a firm-specific markup μ_k so that

$$p_{ikt}^{\text{S}} = \frac{\mu_k \left(1 + \frac{1}{\kappa}\right)}{A_{ikt}},$$

   which gives us the stair-formed supply function

$$S_{it}(P_{it}) = \sum_{j=1}^{n_t} p_{ikt}^{\text{S}} L_{kt}^{\text{rest}}, \tag{19}$$

   where L_{kt}^{rest} is the quantity of labour that has not been bound by contracts in other intermediate markets.
   d. *The individual intermediate market* finds a price P_{it} that is determined by the marginal supplier and buyer. The determination of the price involves several details that will not be covered here. The outcomes of the market process are contracts that are fulfilled within each period, so that the intermediate goods can be used as inputs for the final good production in that period. Similarly, payments are made so that the intermediate good suppliers can fulfil their end-of-period payment of their labour force.
   e. *The overall process in intermediate markets* is governed by an algorithm that determines the sequence in which the intermediate good markets are dealt with in the market process. This sequence is determined by the distance between the highest reservation price and the lowest supply price in the different intermediate good markets. The market with the largest distance is first brought into function. The sequence determined the residual labour L_{kt}^{rest} with which each firm enters a particular intermediate market.

MNW-II. The firm's production and profit:

1. *The basic division of labour* between production activities and research activities is like in LNW-II on page 12, i.e. the firms spend $(1 - r_j)L_{jt}$ units of labour on production and $r_j L_{jt}$ units of labour on research.

2. *Production functions*:
   a. *Production functions for intermediate goods* are like the production function for final output in LNW-II on page 12, i.e.

$$q_{ijt} = A_{ijt} L^{\text{prod}}_{ijt}, \tag{20}$$

   where L_{ijt} for sellers is determined by intermediate good contracts, while buyers of intermediate goods take into account their contracts before they determine the allocation of labour across the different activities.
   b. *Total input of intermediate goods*: To the self-produced quantity of any intermediate good must be added the purchased quantity. Thus the total available amount of i for firm j is

$$Q_{ijt} = q_{ijt} + \sum_{k=1}^{n_t} X_{ijkt}. \tag{21}$$

   c. *Leontief production function for final output*: If the firm produces final output, it does so according to a Leontief production function that for one unit of final output requires a certain amount of labour combined with one unit of each of the $m_{jt} - 1$ units of intermediate goods (cf. equation 1 on page 9). The required number of intermediate goods is determined by the firm's individual structure of production (see MNW-III.3). Thus we have

$$Q_{jt} = \min(A_{1jt} L^{\text{prod}}_{1jt}, \underbrace{Q_{2jt}, \ldots, Q_{ijt}, \ldots, Q_{m_{jt}jt}}_{m_{jt}-1 \text{ intermediate goods}}), \tag{22}$$

   so an important task of a final good producer is obviously to ensure that the mix of in-house production and purchased goods provides exactly the amounts for all inputs to the final output.
3. *Production costs* is like in LNW-II on page 12 for pure intermediate good suppliers. For final good producers the costs of purchasing intermediate goods must be added.
4. *Profit*: Firm j obtains the profit

$$\pi_{jt} = P_{1t} Q_{1jt} + \sum_{i=2}^{m_{jt}} P_{it} (1-\kappa) x_{ijt} - L_{jt} - \sum_{i=2}^{m_{jt}} P_{it} X_{ijt}. \tag{23}$$

MNW-III. The firm's R&D outcome is modelled as a four-stage stochastic process. First, it is determined whether an R&D result is obtained, Second, it is determined whether this result concerns structural innovation or an activity-oriented innovation. Third, is is—in the case of an activity-oriented innovation—determined which activity will be improved. Fourth, the new level of productivity is determined.

1. *Probability of a success of the aggregate R&D activities* is like in LNW-III on page 13.
2. *Division of innovative labour*: Each firm has a firm-specific number of $m_{jt} + 1$ research activities. The allocation of research labour across these activities takes place according to fixed decision rules. The first rule is to use $(1 - \rho_{j)} L^{\text{res}}_{jt}$ researchers for structural research $\rho_j L^{\text{res}}_{jt}$ researchers for activity-specific research. In simplified versions of the MNW models, ρ_j for all firms.
3. *The structural research activity* is switched off in simplified versions of the MNW model. It is, however, an important feature for the study of the emergence of complex economies (see e.g. box 3).
   a. The structural R&D activity (#0) uses labour ($L^{res}_{0jt} = (1 - \rho_{j)} L^{\text{res}}_{jt}$) and provides a more detailed decomposition of the firm's process of production. This decomposition takes the form of an increase in the firm's number of intermediate good activities. If structural R&D has one success in period t, the result is the firm's number of intermediate good activities in the next period is incremented by one, $m_{j,t+1} = m_{jt} + 1$.

b. The outcome of a structural decomposition is neutral with respect to the overall productivity of an autarkic firm. Since unit costs of final output before the change was $\sum 1/A_{ijt}$, the new productivities are

$$A_{ij,t+1} = \begin{cases} \frac{m_{jt}+1}{m_{jt}} A_{ijt}, & \text{for } i \text{ from 1 to } m_{jt} \\ \frac{1}{(1-\frac{m_{jt}}{m_{jt}+1})\sum_{i=1}^{m_{jt}} \frac{1}{A_{ijt}}}, & \text{for } i = m_{jt}+1. \end{cases} \tag{24}$$

4. *The production-activity-specific R&D activities*:
   a. *Methods of research* are like in LNW-III on page 13. For simplicity the following discussion concentrates on the method of cumulative productivity improvement based on the firm's own previous knowledge.
   b. *Specialisation of research*: For each production activity the firm has one R&D activity that is engaged in process innovation and imitation. Thus there are m_{jt} production-activity-specific R&D activities ($\#1,\ldots,\#m_{jt}$) that use labour (L_{ijt}^{res}) and provide improvements in the knowledge for the activities of production; if one of the research activities has a success in period t, the result is that the corresponding production activity has an improvement in its productivity in the next period, $A_{ij,t+1} > A_{ijt}$.
   c. *Allocation of researchers for the production-activity-specific R&D activities* takes place according to fixed rules. One typical rule is to allocate the researchers evenly across the R&D activities. Another rule is to allocate researchers in the same proportions as the production workers. Further information is found in section 4.5.
   d. *Distribution of research outcomes across activities* takes place so that the probability that a particular innovation takes place in R&D activity i is simply the share of researchers engaged in that activity.
   e. *Outcomes of research* are modelled like in LNW-III on page 13.
   f. *Productivity change* is like in LNW-III on page 13.

MNW-IV. The firm's labour accumulation and fissions:

1. *Labour accumulation* is like in LNW-IV.1 on page 13.
2. *Fissions* are like in LNW-IV.2 on page 13. In simplified versions of the MNW model these fissions are switched off.

MNW-V. Main statistics require major additions to LNW-V on page 14. Presently we shall only consider a few statistics.

1–5. The statistics of LNW-V on page 14 can—with several difficulties—be adapted to deal with the MNW model's distribution of labour across firms and its production of final output.

6. The firms's *production shares for individual goods*:

$$s_{ijt}^{\text{quant}} = \frac{Q_{ijt}}{\sum_k Q_{ikt}}.$$

7. *Sectoral employment shares*:

$$s_{it}^{\text{labour}} = \frac{\sum_i \sum_j L_{jt}}{L}.$$

8. *Inverse Herfindahl concentration index for general employment*:

$$H_t^{\text{invL}} = \frac{1}{\sum_{j=1}^{n_t} (s_{jt}^{\text{labour}})^2}.$$

9. *Inverse Herfindahl concentration index for activity-specific employment*:

$$H_{it}^{\text{invL}} = \frac{1}{\sum_{j=1}^{n_t} (s_{ijt}^{\text{labour}})^2}.$$

10. *Roundaboutness index for general employment*:

$$L_t^{\text{dirshare}} = \frac{\sum_{j=1}^{n_t} L_{1jt}}{\sum_{j=1}^{n_t} \sum_{i=2}^{m_t^{\max}} L_{ijt}}.$$

11. *Trade share in intermediate goods*:

$$X_t^{\text{share}} = \frac{\sum_{j=1}^{n_t} \sum_{i=2}^{m_t^{\max}} X_{ijt}}{\sum_{j=1}^{n_t} \sum_{j=2}^{m_t} q_{ijt}}.$$

12. *Overall static inefficiency index*:

$$Q_t^{\text{multigap}} = 1 - \frac{Q_t}{L \frac{1}{\sum_i^{m_t^{\max}} 1/A_{it}^{\max}}}.$$

4.4 Exchange in the MNW model

The above sample of the statistics used to explore the behaviour of the MNW model (MNW-V) gives a first impression of the ways in which this model differs from the standard NW model as well as the LNW model. The main emphasis is on the distribution of labour across the different activities. We may, for instance, ask how labour is distributed between final good production and the production of the intermediate goods that are used as inputs in the production of the final good. We may also ask for the relation between in-house production of intermediate goods and intermediate goods produced for the market. The possibility of making such questions indicates that the MNW model has not only introduced a simple input–output structure but has also endogenised the borderline between the "sectors" of production. In this way the MNW models differs from other sectoral models of the NW model family, like the two-sector models presented by Gerybadze (1982, Ch. 5) and Chiaromonte and Dosi (1993) as well as the multisectoral model of Verspagen (1993, Ch. 7).

Within the framework of the present paper it is impossible to give a full analysis of the MNW model as it is specified above. Instead we shall explore some of the basic characteristics of the model, and here it is convenient to start from the distinction between the final good market and the intermediate good markets. The main thing to understand is that these markets are radically different. This difference will both be discussed in general and illustrated by a simple MNW computer simulation with 20 firms that are engaged in both a final good activity and one intermediate good activity. To simplify further, we let the evolution take place in a situation where transaction costs are so high that there is no trade. Then we stop the simulation and ask what will happen if transaction costs at that point of time is reduced to zero. The results are presented in figure 6

The final good market is depicted in part A of figure 6. It is modelled just as in the NW model and the LNW model. This means that firms produce as much as their capacities allow, while the consumers pays a given amount of money (all their income) for this output. Thus the final good market shows unitary elasticity of demand. This price function is the same in both subfigures—the thick curve. The simulation has been started with stochastically distributed productivities across the 20 firms that all have the same employment of labour. For each firm we find the aggregate productivity, i.e. how much output can be produced by one worker given that this worker also has to take care of the necessary intermediate input and the related R&D. Then we find the aggregate unit costs c_t, which is simply the inverse of the productivity. These unit costs has to be compared with the market price for the final good. For this purpose we construct a long-term supply schedule by taking the firms in ascending order according to their unit costs. The first part of the medium curves in figure 6 represent the firm with the lowest unit costs and its length is the capacity of this firm. Then comes the second firm, and so on to the twentieth firm.

The medium curve that we have constructed does not influence output in the short run. This output is simply the sum of the capacities of all the firms. This means that in panel A.1 firms produce about 15.6 units of output at a price about 6.4—as indicated by the thin lines. This price divided the firms into two classes. Those that have unit costs below 6.4 will have positive profits and thus they

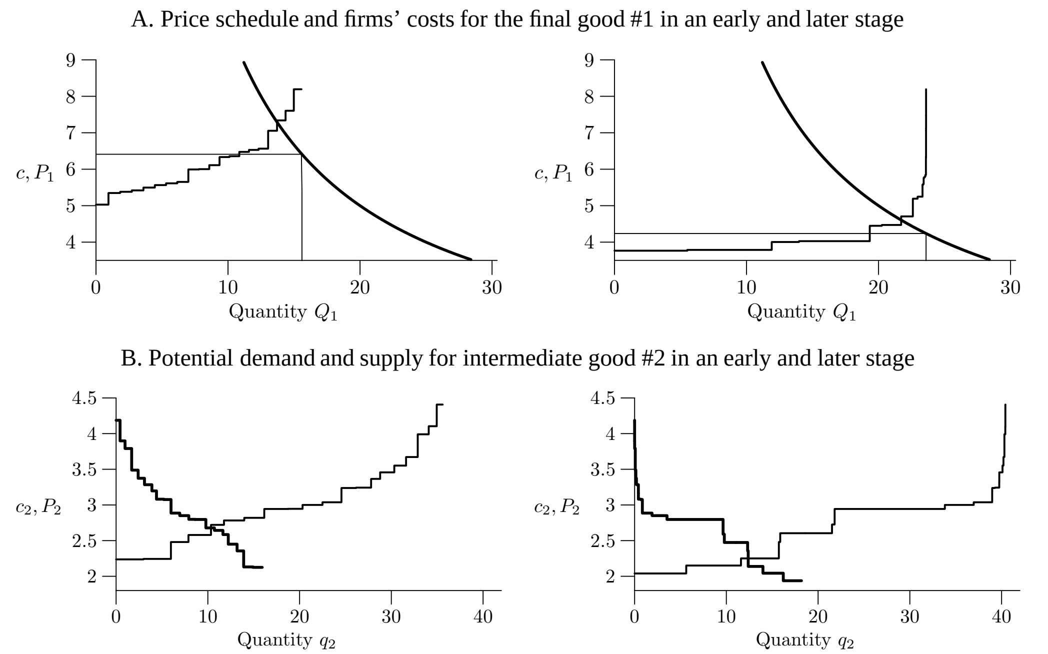

Fig. 6 A comparison of between (A) the final good market #1 and (B) a potential intermediate good market #2. Thick lines deal with demand and the medium lines with supply. The thin lines of (A) shows the market price under the conditions of no intermediate good trade, a capacity-determined supply, and a market clearing through a price set by demand with unitary elasticity. Panels B show the room that is available for mutually profitable trade for the intermediate good.

will expand their labour force. Those that have unit costs above 6.4 will have negative profits and contract their labour force. Even if unit costs were fixed, we would thus over time see a movement of the supply schedule so that the profitable firms would expand their capacity and increase aggregate output, so that marginal firms would become unprofitable. This is one of the reasons for the shift of the supply schedule from panel A. 1 to panel A.2, where we see (1) that the price is lowered, (2) that the profitable firms have obtained large capacities, and (3) that many of the unprofitable firms have contracted to a neglible capacity. There is, however, another reason for the shift, which is obvious from the fact that the profitable firms have significantly lowered their unit costs. This is, of course, that the firms have been performing innovations. Since the larger firms have larger-scale R&D activities, they also show the largest productivity advances.

The potential intermediate good market of panels B have a rather different interpretation. Here we are not yet dealing with a functioning market but rather with the possibilities for such a market. The thick lines represent the potential demand schedules. In panel B.1 we have, like in panel A.1, firms with fairly equal capacities, so the size of their potential demanded quantities do not differ much. There is, however, substantial differences between the costs they can spare by getting rid of the labour they uses for the intermediate good activity. Thus their reservation prices range from about 4.4 to 2.2. The quantities that are potentially demanded is as in equation 18 of the MNW specification, i.e. the quantity of the final good produced in the last period. Since there has not yet been opened up for intermediate trade, all firms are represented on the demand side. Similarly, all firms are potential suppliers of the intermediate good. If they become fully specialised in intermediate production, they use their whole labour force (except the researchers) for this purpose and they can supply their whole production. Therefore, the overall size of the potential supply is—in the two-activity case—about the double of the demand. It is, however, obvious from the figure that only three firms can enter into mutually profitable exchanges with the potential buyers.

Because of labour accumulation and R&D, the demand schedule and the supply schedule will change over time. This is even the case if no trade was introduced in the early stage. Panel B.2 depicts a later stage where the firms' behaviour has not yet been coordinated and disciplined by

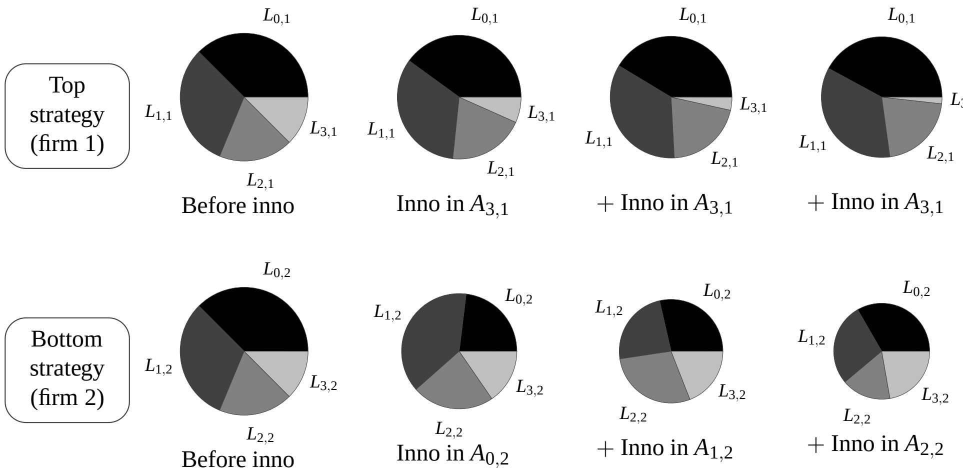

Fig. 7 Examples of the development of in the MNW model of employment shares and total labour costs for two types of strategy when there is no intermediate goods trade. The area of each pie is the labour costs of producing one unit of final output. The slices of the pie are the labour costs in individual activities. [23]

an intermediate good market. Thus it is production and innovation for the final good market that have created the new schedules. On the potential demand side we see that the highest reservation prices comes from firms that, because of their low aggregate productivity, have been reduced to a very small size. There are, however, some profitable firms that have their strength in the final good activity rather than in the intermediate good activity, so that they represent a significant demand. On the supply side, there are three firms that can go into mutually profitable exchanges.

4.5 R&D specialisation in the MNW model

The discussion of exchange in the MNW model has demonstrated the crucial importance of productivity differentials. Like in the classical theory of international trade, there are simply be no exchange of intermediate goods unless there are substantial differences in the demand schedules and supply schedules as defined in equations 18 and 19. The problem is then how these differences arises. In the previous section the differences were produced somewhat artificially. It was simply assumed that the firms randomly chose whether to innovate in the final good activity or the intermediate good activity. There are, however, reasons to believe that this choice will not be made randomly. This is the issue dealt with in figure 7.

Figure 7 depict the strange (but fairly realistic) conditions of autarkic production and the related productivity enhancement under Leontief technology. In the figure we assume that each firm has to produce one unit of each of three intermediate goods and to add a fixed amount of labour to produce one unit of the final good. In this setting we follow a succession of three major innovations that are performed according to two different strategies of R&D specialisation. The first strategy is to emphasise the productivity strengths of the firm and thus to continue to innovate with respect to a (randomly obtained) stronghold. This specialist strategy is called the ‘top strategy’ in the first row of figure 7. The second strategy is to obtain a more or less equilibrated enhancement of the productivities. This generalist strategy is called the ‘bottom strategy’ in the second row of the figure. In both cases the labour shares in the different activities are depicted by pie charts with different shades of grey for each of the four activities, while the area of each pie is the unit labour costs.

The effects of the two strategies becomes immediately clear from the figure. The top strategy serves to innovate in the activity where least labour is spared by each innovation, while the bottom strategy at any point of time focuses on the activity in which most labour can be spared for each innovation. Thus the rule of R&D allocation seems to be clear: focus on the costly areas of production and ignore any tendency to make a follow-up of past successes. A somewhat less efficient strategy, which is however much better than the top strategy and much easier than the bottom strategy, is to allocate researchers in exactly the same proportions as the production workers. These rules are, of course, dependent on the specifications of the MNW model, but they provide good rules of thumb for process innovations.

Unfortunately, there is one problem with these nice rules. This problem is that if they were followed strictly, and if innovations could take place in sufficiently small increments, the strategies would undermine the possibility of moving from autarky to trade in intermediate goods. Instead we recognise easily that the top strategy is the best and fastest way of promoting the emergence of trade. The shift from the bottom strategy or the production-oriented strategy to the top strategy is, however, not easy—neither at an early stage of development or for large and complex firms. One problem is that rules of thumb becomes deeply engrained in organisations and larger social structures. To see this it is useful (and realistic) to think of a large firm whose many different activities are taking place in organisationally separate departments and plants. Each of these departments have their specialised activity in the production of an intermediate good or an intermediate services, according to the rules of Leontief technology. The easiest way of upholding an organisational truce (cf. Nelson and Winter 1982, 107 ff.) between these departments is to have a more or less balanced productivity advance for all departments. This is the major background for what looks like a slowly improving "circular flow" (p. 98) of large firms. But the result of the resultant all-round R&D strategy is that these firms become poorly suited for participating in intermediate goods exchange. Thus we seem to have found an endogeneous reason for the limits of the march toward monopoly and decreasing diversity.

In developing countries there are further reasons for the disencouraging results of the specialising top strategy. Here we not only find vested interests against major changes but also a well-founded scepticism against intermediate supplies that are not adapted to the circumstances and that might not be sufficiently sustainable (cf. the discussion starting with Hirschman 1961, Hirschman 1971). Furthermore, there are high and oscillating transaction costs. So under such conditions it is wise to uphold a broad (although not advanced) in-house competence in many production activities. Unfortunately, this wisdom often leads to vicious circles. In such a context the MNW model gives no easy suggestions. On the contrary, it demonstrates that the emergence of economically coordinated R&D strategies takes place through a difficult and turbulent process.

4.6 The evolving multisectoral economy

The MNW model can be interpreted as a multisectoral growth model with (1) a household sector that sells labour and buys final goods, (2) a final good sector that buys labour and intermediate goods and sells final goods and (3) a maximum of $m_t^{\max} - 1$ intermediate good sectors that buy labour and sell intermediate goods. But because the MNW model takes its starting point in multiactivity firms and excludes any fixed sector boundaries, it is a multisectoral model of a rather special kind. This can be seen in different ways, but the most basic one is, perhaps, seen in relation to the MNW model's handling of what in section 2.3 (starting on page 8) was called the paradox of Leontief technology.

It is obvious from equation 22 on page 8 that the MNW model builds squarely on Leontief technology. We can also construct simple and evolving input–output tables from simulation runs with the MNW model. But nevertheless the MNW model does not show the kind of 'knife-edge' problems that otherwise characterise this realistic type of technology. The reason is, of course, that no firm is entirely dependent on the intermediate supplies of other firms. For instance, if supplies are vanishing because of a sudden increase in transaction costs, production will go on through a changed division of labour between and within firms. The firm may even continue for some time (in a shrinking manner) if it is pushed out on the intermediate good markets and performs badly with respect to the final good. These properties would be even more prominent if we added some flexibility in the wage level for a firm's labour force. But it must be underlined that the tractability of the MNW model is heavily dependent on simplifying assumptions and not least those that relate to the labour market and the homogeneity of wages, labour qualifications, etc.

When we study the long-term evolution of the MNW model's multisectoral economy, it becomes clear that it does not provide a full-blown solution to the paradox of Leontief technology. The problems of the MNW model become most clear if we somehow (e.g. by limiting the efficiency of the

R&D strategies of large firms, by introducing frequent fissions of firms, etc.) obtain a relatively stable selection environment. In such an environment we will see many full specialisations in particular activities (so-called corner solutions), so that highly specialised R&D strategies will become profitable. This means that each firm becomes highly inefficient in producing outside its current activity portfolio. So if supplies of a particular input for some reason is discontinued, the firm will suffer a major set-back and disappear quickly (unless all the other firms have the same problem).

Luckily there are several reasons why this scenario is rather unlikely. The most important is that life as an intermediate good supplier can be quite harsh. Even a position in the lucky end of the supply schedule (cf. panel B of figure 6) is by no means a steady one. First, because of the potentially substantial profits, the strongest firm can outperform the other suppliers and in the end grow so large that it has to take up an additional activity. To be prepared for this eventuality means to apply a R&D strategy that goes beyond the core competence. Second, the potentially quite profitable ability to introduce new intermediate goods presupposes a broad range of competiecies (cf. equation 24). This gives a certain advantage to large firms with a wide scope of activities. Third, there is always the risk that another firm makes a huge productivity increase that in relatively short time pushes a firm out of its stronghold and into other activities. This gives yet another reason for a fairly broad R&D strategy.

Any systematic treatment of these and other issues of the evolving multisectoral economy presupposes analytical work and, especially, simple and systematic computer simulation exercises that bring us beyond the limits of the present paper. But one point should be made in the present context: Although the MNW model suggest solutions to a number of issues that seem to be intrinsic to the Nelson–Winter tradition of studying industrial dynamics, its major advantage is probably found when we turn to broader and more interdisciplinary issues. Somewhat provocatively it has already been pointed out that the paper's LNW model seems to point to "stone age economics" (see box 2 on page 21). In box 3 is given some ideas of using the MNW model for the same purpose.

5 Summary and conclusions

The starting point of this paper was a discussion on the problems and further development of the Nelson–Winter tradition of evolutionary economic modelling and simulation twenty years after *An Evolutionary Theory of Economic Change*. Two strategies were suggested in face of the tendency to decreasing returns of the first generation of evolutionary NW models. The first was the history-friendly modelling strategy proposed by Malerba and Orsenigo as well as Nelson and Winter. However, this strategy fits well into a stronghold of the Nelson–Winter tradition and other neo-Schumpeterian research: the close relationship with empirically oriented researchers of technical change and industrial dynamics. This strategy does not directly confront the inner modelling problems of the Nelson–Winter tradition. Among these problems are a tendency to produce models that depend on too many parameters and the related problem of a lack of a really cumulative tradition of modelling and simulation work, where researchers stand on each other's shoulders. More fundamentally, the issues of diversity and monopolies seem to have been tackled in a too ad hoc manner. Similarly, there has been little discussion of the kind of technology that underlies the models of the Nelson–Winter tradition: evolving Leontief technology. In the context of growth theory this technology has lead to many difficulties because of its in-build 'knife-edge' problems, but this issue has largely been ignored by the NW models. For these and other reasons the paper suggested as a complement to the history-friendly strategy's concrete approach a generalising strategy that is explicitly designed to cope with major modelling and simulation problems. It was also suggested that such a generalising strategy might be quite helpful even when the problem is to deal with history-related issues. Both the theoretical and empirical issues were specified in relation to the particular generalising strategy of the paper, namely to include multiactivity firms and multisectoral evolution into the Nelson–Winter tradition. Apart from confronting some of the problems of NW modelling, the introduction of multiactivity firms might also help to bridge the gap in the *Evolutionary Theory* between the book's organisation-theoretic foundations and its concrete models of growth and industrial dynamics.

Box 3 "Stone age economics" in a MNW model.

Adam Smith (1976[1776]) described division of labour and the propensity to barter as basic characteristics of the human species, but he did not systematically use these assumptions in his broad outline of the stages in the evolution of the human economy. Since the formalisation of economic analysis by Marshall and others, the hope for an integrated treatment of human economic evolution has been pushed aside. The reason for this is mainly to be found in the analytical distinction between households as units of pure consumption and firms as units of pure production. Even Becker's (1991) theory of household production upholds the distinction between economic goods and the household modification of these goods. The problem with the distinction is that Smith's idea of self-sufficient producer–consumer units disappears from the analysis (except in the surviving classical parts of international economics). The MNW model is on the contrary designed to allow for autarky. In its present version this is only possible with respect to intermediate goods, but this property can be extended to final goods as it was suggested in box 2 on page 21, where the final good became the input to the reproduction of humans under 'stone age' conditions. Given this interpretation of the LNW model, it is not difficult to see how a similarly modified MNW model can be used to explore human economic evolution under the conditions of complex K-selection.

The reinterpreted MNW model allows us to start with a single human population that recursively splits up through fissions. Let us assume that the new populations are initially isolated and that they decompose their production activities through random drift.[24] In population j there is a need of one unit of each of the m_{jt} intermediate goods as well as additional labour in order to reproduce a human. In this population it is rational to make balanced progress of all activity-specific productivities (see figure 7), and this fact may to some extent explain the conservatism of traditional economies against exceptional productivities in particular areas.[25] Thus the productivity differences that make inter-population exchange profitable will not emerge easily. Since there are huge transaction costs under stone age conditions, it may take quite long time before systematic exchange emerges. But as soon as the advantages of exchange have been discovered, the situation changes dramatically. This change can of course be due to rational considerations, but it is more likely that exchanging populations grow faster than non-exchanging populations. Later some of the unsuccessful populations will imitate the behaviour of the successful ones.

A major problem with the MNW model is that transaction costs are determined by an exogenous parameter. To reflect the evolution of "stone age economics" there is an obvious need to include innovations with respect to transaction technology, like the emergence and evolution of the institutions of markets and money. From a modelling viewpoint the problem is that we tend to think of transaction innovations in terms of huge jumps ranging from the exchange networks along Australian "Dreaming tracks" (Mulvaney and Kamminga 1999, Ch. 6) and the multi-population summer trading camps of the inuits/eskimos of the Arctic (Burch 1988, 38) to the institutionalised fairs of Medieval Europe (Milgrom, North and Weingast 1991) and the transaction cost part of the Industrial Revolution (North 1981, Ch. 12). It is, however, not difficult to think in terms of incremental transaction innovations that first arise in individual markets (financed by the sellers) and then spill over to the general transaction technology.

The generalising strategy of the paper was developed in two steps. First, the Nelson–Winter model was discussed and transformed into a pure labour model of evolutionary dynamics (the LNW model). Second, the paper included into the LNW model the existence of multiactivity firms, R&D specialisation, and the possibility of exchange in intermediate good markets. The result was the MNW model. The procedure chosen in the paper is based on the conviction that long-term analytical progress seldom comes through gradual extensions to a given model. Sometimes it is also necessary to make simplifications to provide a manageable starting point for an extended set of applications. The stepwise design of the paper also had a pedagogical aim. The intrinsic problems of the Nelson–Winter tradition has made it difficult to teach the tradition, even to advanced students. The design of the basic model and the development of a sequence of more and more complex models might help to change this situation and thus prepare the way for some sort of textbook exposition that is seriously needed in the Nelson–Winter tradition.

The stepwise exposition of the paper started with a brief summary of the basic Nelson–Winter model, and this summary to confront the fact that Nelson and Winter's rich modelling style and the nature of the process of economic evolution exclude the existence of a fully canonical model. But although even the Nelson–Winter tradition in the narrow sense includes a multiplicity of models (formal models that elucidate certain points, a growth model, and a series of models of Schumpeterian competition), it quickly became clear that Nelson and Winter's models of industrial dynamics include a core that has for most purposes become *the* Nelson–Winter model. This model excludes the Leontief technology paradox was dealt with in the Nelson–Winter growth model and it resolves the tendency toward monopoly in a somewhat ad hoc manner. Nevertheless, the standard model has focussed attention on a large number of research topics and has motivated the construction of a significant number novel concepts that are very helpful for the analysis of industrial evolution. So this model provides an important basis for further work. But according to the present paper the model has an important drawback that e.g. makes difficult to really bridge the gap between organisational issues and issues of population-level industrial dynamics. This drawback is the model's heavy dependence upon physical capital in the definition of a firm's capacity and as a focal point for most of the firm's decisions. The paper suggested a radical solution to this problem: to abolish physical capital from the generalised model. This decision lead to the LNW model of an evolving economy where the primary factors of production are labour and knowledge.

The LNW model is primarily a fairly standard Nelson–Winter model, but its construction immediately suggested several changes. First of all, the LNW model is basically a growth model that can be specialised to cover partial processes of industrial dynamics. Second, the concentration on labour and knowledge led to a explicit treatment of research as one of the firm's activities in line with the production activity/activities. Third, the concentration of labour led to an emphasis on organisational issues, e.g. in the question of how new firms emerge. In the Nelson–Winter model this emergence has a parametric character while the LNW model suggests quite another solution: new firms emerge by fissions of old firms. Although it was not included in the specification of the LNW model, these fissions can fairly easily be endogenised in a plausible manner. Fourth, the LNW model got rid of some of the other parameters of the Nelson–Winter model, thus obtaining a higher degree of endogenisation of the elements of the evolutionary process. The design of the LNW model as a general rather than a partial coordination model plays an important role in this result. Fifth, the relative simplicity of the LNW model suggested that Nelson and Winter's split between specialised formal models and complex simulation models is not always necessary. Thus it was shown that the fixed-technology version of the LNW model implements a pure replicator dynamics and thus is covered by a rich set of mathematical theorems that have been developed by evolutionary biologists and evolutionary game theorists. At the same time the replicator dynamics served to introduce the relationship between evolutionary processes and basic laws from probability theory and statistics (which is not so surprising when we remember that R. A. Fisher was a core inventor in both areas). Sixth, the LNW model makes it relatively easy to perform computer simulations and related analytical work in order to explore the conditions for the creation of knowledge as well as imitative behaviour.

In these ways the LNW model was presented as if it was a goal of the generalising strategy to provide such a model. But this is not really the case. Instead the LNW model is intended to function as a stepping stone in the construction of a multiactivity model in the Nelson–Winter family, the MNW model. Given the LNW model, the core analytical step in the construction of the MNW model is really quite simple: Instead of considering all the activities of a firm as an aggregate for which innovations and imitations are performed in a single step, the MNW model splits up this aggregate activity into a number of subactivities that have their own productivities and their related R&D activities. In this way the MNW model tries to relate to Nelson and Winter's organisation-theoretic foundation in a concrete but, admittedly, highly simplified way. The decomposition into multiple activities of production and research was the starting point of a number of extensions of the LNW model. First, the fact that the productivities for the production activities can be improved individually makes in practice each firm unique—even in the case where there are no differences between the firms' aggregate productivities. The reason is, of course, that it is unlikely that the stochastic imporvement of a large number of activities will not take place in the same way in different firms. Second, the existence of multiple activities led the paper into a discussion of how these activities are related. The MNW solution is to have a Leontief production function for the final good activity that includes labour, knowledge, and intermediate goods. Thus the MNW model in a sense reintroduces capital—although in the simplified form of inputs that are used up in each period. Third, the multi-activity approach not only suggested the existence of specialised R&D activities in relation to each production activity but also the existence of structural R&D that serves to decompose the existing activities of the firm. To avoid the complexities of previous models of the author, the structural research was treated in a simplified manner. Nevertheless, it allows an endogeneous evolution of an economic system that starts with the single-activity firms of the LNW model and gradually creates a more and more complex system. Fourth, the MNW model includes the endogenous emergence of intermediate good markets. The starting point is here the productivity differentials that spontaneously emerges through the firms' R&D activities. These differences function like the dynamics comparative advantages of international trade theory. If transaction costs are not prohibitively high, the productivity differences will sooner or later make trade in intermediate goods mutually advantageous for at least some of the firms. Fifth, the organisation of intermediate good markets is a question in itself. The author has previously tried to explore them by means of barter trade that makes as few institutional assumptions as possible. However, in order to cope with more complex economies, the MNW model includes a single price of each intermediate good as well as an algorithm for the process by which all the intermediate good markets function in concert. Sixth, the issue of trade in intermediate goods suggested a renewed discussion of the problem of R&D specialisation. It was shown that for autarkic firms it is rational to use a variant of the generalist R&D strategy. However, this strategy function as a brake on the emergence of productivity differentials that are sufficiently large to allow for wide-spread exchange in intermediate goods. Therefore, alternative, more or less specialised R&D strategies were suggested.

The MNW model is obviously much richer and much more difficult to handle than the LNW model, so it was emphasised that a full analysis and the further development of the MNW model is far beyond the limits of the present type of paper. Still some suggestions on how to explore and develop the MNW model were made. First, there is the problem of the existence of intrafirm conflict and truce as it has been discussed by Nelson and Winter. The MNW model suggests that these conflicts are partly solved by means of truce with respect to R&D strategies. In large firms such a truce will probably tend to promote rather generalist R&D strategies and thus in some ways make it difficult for such firms to exploit fully the advantages of intermediate good trade and the related economy-wide division of labour with respect to knowledge creation. This may to some extent explain why there is a brake on the tendency toward monopoly. Second, there is the paradox of Leontief technology. Here the MNW model suggests a straightforward answer. It is changes in the interfirm division of labour that provide flexibility and avoid knife-edge problems. This solution, of course, is provided at a cost—that some activities in the economy are sometimes performed at highly inefficient costs levels. But this is probably a necessity for making a complex economy function. Third, the problem of the long-term evolution was shortly dealt with. The problem is whether the economy will gradually become less flexible and more likely to encounter Leontief technology-type problems because of the emergence of firms whose competencies are highly specialised. There is no simple answer to this question, but several counter tendencies were pointed at. The basic answer was that life as an intermediate good supplier is much too complex to allow for full R&D specialisation. Fourth, the paper sporadically pointed to the need of evolutionary modellers to handle interdisciplinary questions that have hitherto been beyond the limits of the interests of the researchers of the Nelson–Winter tradition. As an example some suggestions were made in relation to anthropological discussions of "stone age economics".

There is little need to emphasise the obvious limitations of the paper. But a few concluding caveats might be relevant. First, it is not the whole of even a narrowly defined Nelson–Winter tradition that has been taken into account in the present 'generalisation'. One obvious omission is found in the organisation-theoretic part of the *Evolutionary Theory*. This is the discussion of "tacit knowledge". It must be admitted that the author has found in difficult to include this concept into his understanding of the logic of evolutionary analysis, which is nearly by definition about the differential replication of behaviour. Since tacit knowledge is so difficult to replicate, an immediate reaction would be to say that tacit knowledge is part of the phenotypical characteristics and thus not a part of the basic evolutionary process. On second thought there is probably a more important role to play for a sharpened concept of tacit knowledge, but it must be in relation to easily replicable behaviours. Second, the MNW model was intended as a way of moving the focus away from production and process innovation to exchange and product innovation. But this ambition has not been fulfilled by the

MNW model, although it is not difficult to suggest some developments in this direction. One of them is to make the MNW transaction technology subject to R&D based improvements. Another is to allow for idiosyncrasies of the intrafirm decomposition of activities, e.g. by adding randomly chosen indexes of 'mismatches' between firm-specific brands of intermediate goods and the requirements of the production process of the buyers. But the feeling of the author is that such additions might bring more heat than light to the application of the MNW model. Third, there is the related question of how to relate to standard analysis of monopolistic competition of the Chamberlin tradition. Again the answer is ambiguous: There is a need to include a rich demand side and the dynamics of customer markets as they have been explored by theorists as well as marketing researchers. However, it would be nice to do so in the same way as the intermediate markets were introduced in terms of the behaviour of symmetrically defined economic agents. Since a basic ambition of the MNW model is to overcome the split between pure consumers and pure producers, the best way would be to treat both households and firms as entities perform both production and consumption.

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Notes

[1] Nelson and Winter's *Evolutionary Theory* is to a large extent based on a series of important research papers. References to these papers will mainly be made through references to the related chapters of their book. In box 1 on page 5 there is, however, a listing of some of the underlying papers in relation to the different models of the book.

[2] To take up a controversial issue from Nelson's (1995) survey, it is easy enough to criticise the sociobiologists for their lack of a treatment of the "institutional complexity of modern societies" (p. 60), but sociobiologists and other anthropologists find little support for an evolutionary economic treatment of "stone-age economics" in the Nelson–Winter tradition. Thus there seems to be a need of a radical generalisation of the Nelson–Winter model, but it is probably not easy to fulfil the task. The producer–consumer household version of the multiactivity model of the present paper (cf. Andersen 1998, 2001) might give some suggestions on this topic. Even simpler possibilities emerge from the models of the present paper as sketched out in box 2 on page 21 and in box 3 on page 33.

[3] It has repeatedly been emphasised by Nelson and Winter and other neo-Schumeterians that modellers should not get "carried away by evolutionary analogies and … mistake the analogy for reality, as often occurs with computer simulation models" (Freeman 1992, 123). In this respect they belong to a longer Schumpeterian tradition that can be traced back to Schumpeter who in his last published paper states that "what is really required is a large collection of industrial and locational monographs". These monographs should give "proper attention on the one hand to the incessant historical change in the production and consumption functions and on the other hand to the quality and behavior of the leading personnel" (Schumpeter 1951, 314). The idea is apparently to create a new set of 'stylised facts' that e.g. can help to focus the analysis of theorists on the really important aspects of economic evolution. The works of e.g. Chandler (1962), Freeman (1974) and Rosenberg (1976) fit well into this programme. But the goal of developing general theories and models easily can be pushed aside in this Schumpeterian programme. Thus Nelson and Winter (1982, 48) point out that researchers engaged in the programme "can be accused of not appreciating the importance of a coherent theoretical structure". Nelson and Winter (1982, 45–48), however, continue the Schumpeterian tradition by emphasising that the much-needed theoretical models have to be developed in tandem with "appreciative theorising" that relates directly to historical and statistical evidence.

[4]Generalisation is a basic tool in mathematics, and here it is obvious that e.g. the generalised theory of continuous functions for many purposes is not nearly as interesting as the detailed theories like that of the special case of $y = e^x$.

[5]The fact that a generalised theory is not generally better than a specialised theory can immediately be recognised when we consider the problems of usability. Here we see that although a general model includes the specialised model as a special case, it seldom does so in an easy way. So it is obvious that a generalisation is not an unconditional advantage.

[6]Lakatos (1970) and Blaug (1980) treat rather extensively the process of incremental generalisation within "scientific research programmes", but they appear to overdramatise the issue of breaking out of an existing research programme.

[7]Most of the NW models are not very well documented. An effort of reconstructing the NW computer simulation models is recorded in Andersen, Jensen, Madsen and Jørgensen (1996). These reconstructions apply the programming language of the Maple system for symbolic (and numerical) mathematics. Recently the conditions for such reconstructive work has improved because of the Laboratory for Simulation Development (Lsd). Lsd is an open source software project whose results are freely available for both UNIX/Linux systems and MS Windows systems. It was developed by Marco Valente at IIASA in Laxemburg, at DRUID in Aalborg, and at the University of Trento as a general system for both ordinary users and advanced programmers dealing with evolutionary modelling. How the standard NW model can be represented in this system is described in Valente and Andersen (forthcoming). The XNW99 model (which might after its authors also be called the MNOW model) will, hopefully, soon be publicly available as an Lsd simulation model. Alternative simulation tools are presented by Gilbert and Troitzsch (1999) and the systems dynamics tradition is explored in great detail by Sterman (2000).

[8]A more extensive overview over the standard NW model is found in Winter (1984); see also Andersen (1996b, Ch. 4).

[9]In the opinion of the author structure diagrams and flowcharts are important tools for the development and presentation of evolutionary models. Unfortunately, they are not easy to modify and mix with mathematics unless you have a powerful drawing tool. The author applies the LATEX system for programmable typesetting and has based his programs and figures on the LATEX 2_ε packages `pstcol` and `pst-node`, cf. Goossens, Rahtz and Mittelbach (1997, Ch. 4, especially 121–125).

[10]The rules of capital accumulation in the standard NW model are (in a mathematical notation that does not fully correspond to the notation used in the rest of the paper):

1. In each period the firm's capital is depreciated by the general rate of depreciation (δ). The firm cannot decrease its capacity faster than a the rate determined by depreciation. On the other hand, the firm's expansion of its productive capacity has to start by compensating for depreciation.
2. The financial constraint per unit of capital ($I_{jt}^{\max}$) is determined by the profit rate per unit of capital (π_{jt}). If there are no positive profits, no investment can take place. Otherwise, the possible investment per unit of capital is profits times a factor that is influenced both by the firm's internal propensity to invest and the support by the banking system.
3. The desired investment per unit of capital is determined in terms of markup pricing in the following way:
   a. The expected markup of price over costs for the next period is found by comparing the existing price with the unit costs determined by the productivity for the next period, i.e. $\mu_{jt}^{\text{expect}} = \frac{P_t A_{jt}}{C}$.
   b. The target markup of price over costs depends on the market share of the firm (s_{jt}) as it has been studied for the multi-firm version of the Cournot model (see e.g. Scherer and Ross 1990, Ch. 6). In the present case we have unitary elasticity of demand and fixed unit costs, so the target markup factor in the non-evolutionary Cournot case is $\frac{1}{1-s_{jt}}$. This requirement leads to a conservative investment behaviour of large firms and thus puts a brake on the concentration ratio. However, since NW covers an evolutionary situation, it pays for firms to have a more aggressive investment behaviour (and thus a lower target markup). The target markup in the dynamical process is e.g. $\mu_{jt}^{\text{target}} = \frac{\eta}{\eta - s_{jt}}$, where $\eta \geq 1$.
   c. The desired investment per unit of capital (I_{jt}^{desire}) is determined by the ratio between the target markup and the expected markup, i.e. $\mu_{jt}^{\text{target}} / \mu_{jt}^{\text{expect}}$. If this ratio is smaller than 1, $I_{jt}^{\text{desire}} = 0$. Otherwise it is equal to the ratio between target markup and expected markup (plus a compensation for depreciation).

4. The actual investment per unit of capital is, of course, determined by both the financial constraint and the desired investment, i.e. $I_{jt} = \min(I_{jt}^{\max}, I_{jt}^{\text{desire}})$.
5. The capital level for the next period is determined by the present level of capital, the investment per unit of capital and the depreciation per unit of capital, i.e. $K_{j,t+1} = (1 + I_{jt} - \delta)K_{jt}$.

[11]An extensive discussion of how to bias search toward improved labour productivity (or improved capital productivity) is found in Nelson and Winter (1982, Chs. 7 and 9).

[12]This nightmare of knife-edges in a Leontief economy has been implemented into a formal growth model by Pasinetti (1981, 1993). Pasinetti's model demonstrates the huge need for intervention that spring from a non-evolutionary version of Leontief technology.

[13]The omission of physical capital means that we cannot immediately relate to the extensions of the NW model to include vintage capital as it has been developed in Silverberg (1987), Silverberg, Dosi and Orsenigo (1988), and in later papers. However, underlying the LNW model is a replicator dynamics approach that seems to be close to the endeavours of Silverberg.

[14]Some rules are 'hidden' by being set to unity. For instance, the capacity utilisation rule in the production function of the NW model is $u = 1$.

[15]We could, of course, operate with different wage rates for production workers and research workers as in Aghion and Howitt (1998, Ch. 2). But this makes little sense since we shall give no account for the supply of researchers. It would also be against the logic of the present design of the LNW model.

[16]Each firm's employment change is equal to its profits, so aggregate change is equal to aggregate profits. By applying a few of the LNW model's equations, we find that $\sum \pi_{jt} = \sum(P_t Q_{jt} - L_{jt}) = P_t \sum Q_{jt} - \sum L_{jt} = \frac{\sum L_{jt}}{\sum Q_{jt}} \sum Q_{jt} - \sum L_{jt} = 0$.

[17]In the LNW model the selection environment quickly changes, so there is little meaning in treating the research intensity rule as an evolving state variable. Instead experiments start with firms with different rules.

[18]This is only strictly true when the setup of the LNW model is so that we can apply the rule of the additivity of Poisson processes, see e.g. Aghion and Howitt (1998, 55).

[19]NW10.2 (see box 1 on page 5) uses the price of the product as *numéraire* and thus set P to unity. Instead it uses the wage level to weed out under-average performers. This means that unit costs are not constant, and this fact leads to a slightly more realistic but also more cumbersome path to a Fisher-like theorem.

[20]The term 'replicator equation' was first used by Schuster and Sigmund (1983).

[21]If we want to catch the whole of human evolution in some sort of LNW model, we must connect to the genetic evolution of the characteristics of the human species, but it is far beyond the limits of the paper to discuss this issue. For diverse attempts in this highly controversial field see e.g. Lumsden and Wilson (1981), Barkov, Cosmides and Tooby (1992), and Katz (2000).

[22]Young (1928) points out that his idea relates to the concept of varying "roundaboutness" of production in the Austrian school of economics (cf. Andersen 1996a).

[23]Since the use of piecharts is a natural way of depicting aspects of the multiactivity economy of the MNW model, some readers might be interested in knowing a recipe for producing the pies. (1) $area = \sum_{i=1}^{m} 1/A_i$. (2) $radius = \sqrt{area/\pi}$. (3) $slice_i = 360/(A_i * area)$. (4) If you use LaTeX, apply the drawing procedure described by Goossens et al. (1997, 134 f.). (5) If you want to place many pies in an orderly manner like in the figure, ask the author for his code that uses the LaTeX packages `pstcol` and `pst-node`.

[24]Although there are obvious empirical advantages of intra-population decomposition of activities, the MNW model assumes that such a decomposition is neutral with respect to productivity.

[25]An alternative explanation in terms of vested interests is given by e.g. Hirschman (1961, 11 ff.).

References

Aghion, P. and Howitt, P. (1998), *Endogenous Growth Theory*, MIT Press, Cambridge, Mass. and London.

Andersen, E. S. (1996a), The evolution of an industrial sector with a varying degree of roundaboutness of production, *DRUID Working Paper 96-13*, Danish Research Unit for Industrial Dynamics, Copenhagen and Aalborg.
URL: *http://www.business.auc.dk/druid/wp/pdf_files/96-13.pdf*

Andersen, E. S. (1996b), *Evolutionary Economics: Post-Schumpeterian Contributions*, paperback edn, Pinter, London.

Andersen, E. S. (1996c), From static structures to dynamics: Specialisation and innovative linkages, *in* C. DeBresson (ed.), *Economic Interdependence and Innovative Activity: An Input-Output Analysis*, Elgar, Aldershot, pp. 331–353.

Andersen, E. S. (1997), Neo- and post-Schumpeterian contributions to evolutionary economics, *in* J. Reijnders (ed.), *Economics and Evolution*, Elgar, Cheltenham, pp. 109–135.

Andersen, E. S. (1998), The dynamics of the organisation of industry, *DRUID Working Paper 98-13*, Danish Research Unit for Industrial Dynamics, Copenhagen and Aalborg.
URL: *http://www.business.auc.dk/druid/wp/pdf_files/98-13.pdf*

Andersen, E. S. (1999), Multisectoral growth and national innovation systems, *Nordic Journal of Political Economy* **25**, 33–52.

Andersen, E. S. (2001), Satiation in an evolutionary model of structural economic dynamics, *Journal of Evolutionary Economics* **11**, 143–164.

Andersen, E. S., Jensen, A. K., Madsen, L. and Jørgensen, M. (1996), The Nelson and Winter models revisited: Prototypes for computer-based reconstruction of Schumpeterian competition, *DRUID Working Papers 96-5*, Danish Research Unit for Industrial Dynamics, Copenhagen and Aalborg.
URL: *http://www.business.auc.dk/druid/wp/pdf_files/96-5.pdf*

Arthur, W. B. (1989), Competing technologies, increasing returns, and lock-in by historical events, *Economic Journal* **99**, 116–131.

Aversi, R., Dosi, G., Fagiolo, G., Meacci, M. and Olivetti, C. (1999), Demand dynamics with socially evolving preferences, *Industrial and Corporate Change* **8**, 353–408.

Axelrod, R. (1997), Advancing the art of simulation in the social sciences, *in* R. Conte, R. Hegselmann and P. Terna (eds), *Simulating Social Phenomena*, Springer, Berlin, pp. 21–40.

Ballot, G. and Taymaz, E. (1997), The dynamics of firms in a micro-to-macro model: The role of training, learning and innovation, *Journal of Evolutionary Economics* **7**, 435–457.

Barkov, J. B., Cosmides, L. and Tooby, J. (1992), *The Adapted Mind*, Oxford University Press, New York.

Becker, G. S. (1991), *A Treatise on the Family*, enlarged edn, Harvard University Press, Cambridge, Mass. and London.

Blaug, M. (1980), *The Methodology of Economics: Or How Economists Explain*, Cambridge University Press, Cambridge.

Boyd, R. and Richerson, P. J. (1985), *Culture and the Evolutionary Process*, University of Chicago Press, Chicago and London.

Burch, E. S. (1988), *The Eskimos*, University of Oklahoma Press, Norman, Okla. and London.

Chamberlin, E. H. (1962[1933]), *The Theory of Monopolistic Competition: A Re-orientation of the Theory of Value*, Harvard University Press, Cambridge, Mass. and London.

Chandler, A. D. (1962), *Strategy and Structure: Chapters in the History of the Industrial Enterprise*, MIT Press, Cambridge, Mass.

Chiaromonte, F. and Dosi, G. (1993), The micro foundations of competitiveness and their macroeconomic implications, *in* D. Foray and C. Freeman (eds), *Technology and the Wealth of Nations: The Dynamics of Constructed Advantage*, Pinter, London, pp. 107–134.

Dawkins, R. (1989), *The Selfish Gene*, revised edn, Oxford University Press, Oxford and New York.

Dosi, G. and Nelson, R. R. (1994), An introduction to evolutionary theories in economics, *Journal of Evolutionary Economics* **4**, 153–172.

Fisher, R. A. (1999[1930]), *The Genetical Theory of Natural Selection: A Complete Variorum Edition*, Oxford University Press, Oxford.

Freeman, C. (1974), *The Economics of Industrial Innovation*, Penguin, Harmondsworth.

Freeman, C. (1992), Innovation, changes of techno-economic paradigm and biological analogies in economics, *The Economics of Hope: Essays on Technical Change, Economic Growth and the Environment*, Pinter, London and New York, pp. 121–142.

Futuyma, D. J. (1998), *Evolutionary Biology*, 3rd edn, Sinauer, Sunderland, Mass.

Gerybadze, A. (1982), *Innovation, Wettbewerb und Evolution: Eine mikro- und mesoökonomische Untersuchung des Anpassungsprozesses von Herstellern und Anwendern neuer Produzentengüter*, J.C.B. Mohr (Paul Siebeck), Tübingen.

Gilbert, N. and Troitzsch, K. G. (1999), *Simulation for the Social Scientist*, Open University Press, Buckingham and Philadelphia, Penn.

Goossens, M., Rahtz, S. and Mittelbach, F. (1997), *The LaTeX Graphics Companion: Illustrating Documents with TeX and PostScript*, Addison–Wesley, Reading, Mass.

Hirschman, A. O. (1961), *The Strategy of Economic Development*, paperback edn, Yale University Press, New Haven, Conn.

Hirschman, A. O. (1971), *A Bias for Hope: Essays on Development and Latin America*, Yale University Press, New Haven and London.

Hofbauer, J. and Sigmund, K. (1998), *Evolutionary Games and Population Dymanics*, Cambridge University Press, Cambridge.

Kaldor, N. (1981), The role of increasing returns, technical progress and cumulative causation in the theory of international trade and economic growth, *Économie Appliquée* **34**, 593–617.

Katz, L. D. (2000), *Evolutionary Origins of Morality: Cross-Disciplinary Perspectives*, Imprint Academic, Thorverton, UK, and Bowling Green, Ohio.

Kingsland, S. E. (1985), *Modeling Nature: Episodes in the History of Population Ecology*, University of Chicago Press, Chicago and London.

Kwasnicki, W. (2001), Comparative analysis of selected neo-Schumpeterian models of industrial dynamics, *Nelson and Winter Conference, 12-15 June 2001*, DRUID, Aalborg.
URL: *http://www.business.auc.dk/druid/conferences/nw/paper1/Kwasnicki1.pdf*

Lakatos, I. (1970), Falsification and the methodology of scientific research programmes, *in* I. Lakatos and A. Musgrave (eds), *Criticism and the Growth of Knowledge*, Vol. 4 of Proceedings of the International Colloquium in the Philosophy of Science, London, 1965, Cambridge University Press, Cambridge, pp. 91–195.

Lancaster, K. (1979), *Variety, Equity, and Efficiency: Product Variety in an Industrial Society*, Blackwell, Oxford.

Langlois, R. N. and Robertson, P. L. (1995), *Firms, Markets and Economic Change: A Dynamic Theory of Business Institutions*, Routledge, London and New York.

Leijonhufvud, A. (1986), Capitalism and the factory system, *in* R. N. Langlois (ed.), *Economics as a Process: Essays in the New Institutional Economics*, Cambridge University Press, Cambridge, pp. 203–223.

Leontief, W. (1966[1941]), *The Structure of American Economy, 1919-1939: An Empirical Application of Equilibrium Analysis*, Oxford University Press, New York.

Lumsden, C. and Wilson, E. O. (1981), *Genes, Mind and Culture*, Harvard University Press, Cambridge, Mass.

Malerba, F., Nelson, R. R., Orsenigo, L. and Winter, S. G. (1999), "History-friendly" models of industry evolution: The computer industry, *Industrial and Corporate Change* **8**, 1–36.

Marshall, A. (1961[1890]), *Principles of Economics*, 9th (Variorum) edn. with annotations by C. W. Guillebaud, Macmillan, London.

Metcalfe, J. S. (1994), Competition, Fisher's principle and increasing returns in the selection process, *Journal of Evolutionary Economics* **4**, 327–346.

Milgrom, P. R., North, D. C. and Weingast, B. R. (1991), The role of institutions in the revival of trade: The law merchant, private judges, and the Champaigne fairs, *Economics and Politics* **2**, 1–23.

Mulvaney, J. and Kamminga, J. (1999), *Prehistory of Australia*, Allen & Unwin, St. Leonards, NSW.

Nelson, R. R. (1968), A "diffusion model" of international productivity differences in manufacturing industry, *American Economic Review* **58**, 1219–1248.

Nelson, R. R. (1995), Recent evolutionary theorizing about economic change, *Journal of Economic Literature* **33**, 48–90.

Nelson, R. R. and Winter, S. G. (1973), Toward an evolutionary theory of economic capabilities, *American Economic Review* **63**, 440–449.

Nelson, R. R. and Winter, S. G. (1974), Neoclassical vs evolutionary theories of economic growth: Critique and prospectus, *Economic Journal* **84**, 886–905.

Nelson, R. R. and Winter, S. G. (1975), Factor price changes and factor substitution in an evolutionary model, *Bell Journal of Economics* **6**, 466–486.

Nelson, R. R. and Winter, S. G. (1977), Dynamic competition and technical progress, *in* B. Balassa and R. R. Nelson (eds), *Economic Progress, Private Values, and Public Policy: Essays in Honor of William Fellner*, North-Holland, Amsterdam, pp. 57–101.

Nelson, R. R. and Winter, S. G. (1978), Forces generating and limiting concentration under Schumpeterian competition, *Bell Journal of Economics* **9**, 524–548.

Nelson, R. R. and Winter, S. G. (1982), *An Evolutionary Theory of Economic Change*, Belknap Press, Cambridge, Mass. and London.

Nelson, R. R. and Winter, S. G. (1982b), The Schumpeterian tradeoff revisited, *American Economic Review* **72**, 114–132.

Nelson, R. R., Winter, S. G. and Schuette, H. L. (1976), Technical change in an evolutionary model, *Quarterly Journal of Economics* **90**, 90–118.

North, D. C. (1981), *Structure and Change in Economic History*, Norton, New York and London.

Pasinetti, L. L. (1981), *Structural Change and Economic Growth: A Theoretical Essay on the Dynamics of the Wealth of Nations*, Cambridge University Press, Cambridge.

Pasinetti, L. L. (1993), *Structural Economic Dynamics: A Theory of the Economic Consequences of Human Learning*, Cambridge University Press, Cambridge.

Popper, K. R. (1972), *Objective Knowledge: An Evolutionary Approach*, Clarendon, Oxford.

Richardson, G. B. (1975), Adam Smith on competition and increasing returns, *in* A. S. Skinner and T. Wilson (eds), *Essays on Adam Smith*, Clarendon, Oxford, pp. 350–360.

Romer, P. M. (1986), Increasing returns and long-run growth, *Journal of Political Economy* **94**, 1002–1038.

Romer, P. M. (1993), Idea gaps and object gaps in economic development, *Journal of Monetary Economics* **32**, 543–573.

Rosenberg, N. (1976), *Perspectives on Technology*, Cambridge University Press, Cambridge.

Sahlins, M. D. (1974), *Stone Age Economics*, Tavistock, London.

Scherer, F. M. and Ross, D. (1990), *Industrial Market Structure and Economic Performance*, 3rd edn, Houghton Mifflin.

Schumpeter, J. A. (1951), The historical approach to the analysis of business cycles, *Essays on Economic Topics*, ed. R.V. Clemence, Kennikat, Port Washington, N.Y., pp. 308–315.

Schuster, P. and Sigmund, K. (1983), Replicator dynamics, *Journal of Theoretical Biology* **100**, 533–538.

Shackle, G. L. S. (1967), *The Years of High Theory: Invention and Tradition in Economic Thought 1926-1939*, Cambridge University Press, Cambridge.

Silverberg, G. (1987), Technical progress, capital accumulation and effective demand: A self-organisation model, *in* D. Batten, J. Casti and B. Johansson (eds), *Economic Evolution and Structural Adjustment*, Springer, Berlin.

Silverberg, G. (1988), Modelling economic dynamics and technical change: Mathematical approaches to self-organization and evolution, *in* G. Dosi, C. Freeman, R. Nelson, G. Silverberg and L. Soete (eds), *Technical Change and Economic Theory*, Pinter, London, pp. 531–559.

Silverberg, G., Dosi, G. and Orsenigo, L. (1988), Innovation, diversity and diffusion: A self-organization model, *Economic Journal* **98**, 1032–1054.

Silverberg, G. and Verspagen, B. (1994), Collective learning, innovation and growth in a boundedly rational, evolutionary world, *Journal of Evolutionary Economics* **4**, 207–226.

Smith, A. (1976[1776]), *An Inquiry into the Nature and Causes of the Wealth of Nations*, ed. R.H. Cambell and A.S. Skinner, Clarendon, Oxford.

Sober, E. (1984), *The Nature of Selection: Evolutionary Theory in Philosophical Focus*, MIT Press, Cambridge, Mass. and London.

Sraffa, P. (1926), The laws of returns under competitive conditions, *Economic Journal* **36**, 535–550.

Sterman, J. D. (2000), *Business Dynamics: Systems Thinking and Modelling for a Complex World*, McGraw-Hill, Boston, Mass.

Teece, D. J., Rumelt, R., Dosi, G. and Winter, S. G. (1994), Understanding corporate coherence: Theory and evidence, *Journal of Economic Behavior and Organization* **23**, 1–30.

Valente, M. and Andersen, E. S. (forthcoming), A hands-on approach to evolutionary simulation: Nelson and Winter models in the Laboratory for Simulation Development, *Electronic Journal of Evolutionary Modeling and Economic Dynamics* **1**.
URL: *http://www.e-jemed.org/*

Varian, H. R. (1997), How to build an economic model in your spare time, *in* M. Szenberg (ed.), *Passion and Craft: Economists at Work*, University of Michigan Press, Ann Arbor, Mich.

Verspagen, B. (1993), *Uneven Growth Between Interdependent Economies: An Evolutionary View on Technology Gaps, Trade and Growth*, Avebury, Aldershot.

Winter, S. G. (1964), Economic “natural selection” and the theory of the firm, *Yale Economic Essays* **4**, 225–272.

Winter, S. G. (1971), Satisficing, selection and the innovating remnant, *Quarterly Journal of Economics* **85**, 237–261.

Winter, S. G. (1984), Schumpeterian competition in alternative technological regimes, *Journal of Economic Behavior and Organization* **5**, 287–320.

Winter, S. G. (1987), Natural selection and evolution, *in* J. Eatwell, M. Milgate and P. Newman (eds), *The New Palgrave: A Dictionary of Economics*, Vol. 3, Macmillan, London and Basingstoke, pp. 614–617.

Winter, S. G. (1991), On Coase, competence, and the corporation, *in* O. E. Williamson and S. G. Winter (eds), *The Nature of the Firm: Origins, Evolution, and Development*, Oxford University Press, New York and Oxford, pp. 179–195.

Winter, S. G., Kaniovski, Y. M. and Dosi, G. (2000), Modeling industrial dynamics with innovative entrants, *Structural Change and Economic Dynamics* **11**, 255–293.

Yang, X. and Ng, Y.-K. (1993), *Specialization and Economic Organization: A New Classical Microeconomic Framework*, North-Holland, Amsterdam.

Young, A. A. (1928), Increasing returns and economic progress, *Economic Journal* **38**, 527–542.